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A Generalization of the Universality of Power Lacunary Series

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Abstract—In this paper we consider the uniform approximation of a function from a certain class by sums obtained as a result of some matrix transformation of the universal power lacunary series.

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The power series universal in a real domain were first constructed by the Hungarian mathematician Fekete in 1906. Universal trigonometric series were studied by D. E. Men'shov [1] in 1945; the universality of power series on closed sets was studied by A. I. Seleznev [2] in 1951 and C. K. Chui and M. N. Parnes [3] in 1971; V. Luh [4] in 1976 generalized the latter result onto the case of matrix transformations.

In this paper, unlike those mentioned above, we consider a universal power series of the lacunary structure. The main result of the paper is Theorem 3. With the help of matrix transformations it generalizes the universality of a power series for a case of special sums. For an arbitrary function from a certain class on closed sets of a special type we construct sums which approximate this function.

In [5] (P. 193) under the condition

$$\lim_{n\to\infty}\frac{n}{\lambda_n}=\tau<1 \tag{1}$$

A. F. Leont'ev proved the completeness (in the sense of the uniform approximation in the class of analytic functions) of the subsystem

$$\{z^{\lambda_n}\}_{n=1,2,\dots} \tag{2}$$

in the angular domain $D^{(\tau)}$ of opening angle $2\pi\tau$.

We say that an *angular domain of opening angle* φ is described by a Jordan arc $\mathscr{L}$ connecting a point of the circle $|z|=r$, $0<r<\infty$, with $z=\infty$ under rotation of the plane z by the angle φ with respect to the origin $z=0$.

Along with the completeness of system (2), in the domain $D^{(\tau)}$ condition (1) guarantees the completeness of

$$\{z^{\lambda_n}\}_{n=p,p+1,\dots}, \tag{3}$$

where p is an arbitrary natural number. Hence, as is shown in [6], there exists a series

$$\sum_{n=1}^{\infty}c_n z^{\lambda_n} \tag{4}$$

universal in the domain $D^{(\tau)}$. This means that for any function $f\in C_A(F)$, where F is an arbitrary compact set contained in $D^{(\tau)}$, whose complement with respect to the extended complex plane z is a domain, there exists a subsequence of partial sums of series (4) uniformly convergent on F to the function $f(z)$.

We denote by $C_A(F)$ the class of functions that are continuous on F and analytic in a neighborhood of each interior point of this set. The next theorem also follows from [6].

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