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System of Singularly Perturbed Equations with Differential Turning Point of the First Kind

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Abstract—We construct uniform asymptotic of solution to a system of differential equations with small parameter at the higher derivative and turning point. In particular, we consider the cases where the spectrum of boundary operator contains multiple elements and elements identically equaling zero.

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INTRODUCTION

Theory of singularly perturbed differential equations with turning point is presented in numerous works dealing with problems containing a small parameter at the higher derivative. A classic sample is the Liouville equation; it is studied rather well [1, 2]. The equations of third and fourth orders

$$\varepsilon y'''(x,\varepsilon) + x\widetilde{a}(x)y'(x,\varepsilon) + b(x)y(x,\varepsilon) = h(x), \tag{0.1}$$

$$\varepsilon^2 y^{(4)}(x,\varepsilon) + a(x)y^{(2)}(x,\varepsilon) + b(x)y'(x,\varepsilon) + c(x)y(x,\varepsilon) = h(x),$$

are also investigated in a number of papers [3–5]. The development of asymptotical theory for scalar equations of third order begins in the works by R. E. Langer [6, 7] and R. E. Bragg [8]. These authors obtain asymptotic approximations from the both sides of the turning points by terms of detailed study of corresponding integral equations. V. N. Bobochko [9, 10] studied Eq. (0.1) by means of a method developed by him for the Liouville equation [2]. Later this method was generalized for the case of differential turning point. In [9, 10] the author emphasizes the fact that structure of the solution depends essentially on the signs of functions $\widetilde{a}(x)$ and $b(x)$. But compared with the Liouville equation there are still many unresolved issues. In particular, the following problem is unresolved. V. Wasow [1] built uniform asymptotic of solution to a system of differential equations corresponding to the Liouville equation. From the viewpoint of the modern applied mathematics and theoretical physics the issue of particular interest is the study of vector equations equivalent to that scalar ones. The author obtains asymptotical approximations for a system with matrix of dimension 3×3. P. F. Hsieh [11] generalizes results by R. E. Langer, K. K. Lin, Sibuia, and considers equation

$$\varepsilon^2 y''' + R_1 y'' + R_2 y' + R_3 y = zDy' + P(z)y,$$

where y is two-dimensional vector function, ε is complex parameter, $P(z)$ is matrix of dimension 2×2, D is non-degenerated constant diagonal matrix with distinct diagonal elements, components R_h, $h = 1, 2, 3$, are holomorphic with regard to z and to ε in D and Θ. These components allow an expansion in degrees of ε with holomorphic coefficients in D uniformly vanishing for ε tending to zero in Θ. The point $z = 0$ is a turning point for the equation. By means of well-known transformations the equation reduces to the system

$$\varepsilon Y' = M(z,\varepsilon)Y,$$

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