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Special Version of Collocation Method for Integral Equations of the Third Kind With Fixed Singularities in a Kernel

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Abstract—We consider a linear integral equation of the third kind with fixed singularities in its kernel. We offer and prove a special version of collocation method for its approximate solving.

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We study linear integral equation of the third kind with fixed singularities in its kernel (E3KFS)

$$Ax \equiv x(t)\prod_{j=1}^{l}(t-t_j)^{m_j} + \int_{-1}^{1} K(t,s)[(s+1)^{p_1}(1-s)^{p_2}]^{-1}x(s)ds = y(t), \tag{1}$$

where $t \in I \equiv [-1,1]$, $t_j \in (-1,1)$, $m_j \in \mathbb{N}$ $(j=\overline{1,l})$; $p_1, p_2 \in \mathbb{R}^+$, K and y are known continuous functions with certain pointwise "smoothness" properties, $x(t)$ is the desired function, and integral is understood as the Hadamard finite part ([1], pp. 144–150). Equations (1) have extensive applications both in theory and in practice. A number of important problems of elasticity theory, transfer of neutrons, particle scattering (see [2, 3] and references in [3, 4]), and theory of differential equations of mixed type [5] reduces to that equations. As a rule, intrinsic classes of solutions of equations of the third kind are special spaces of distributions (SD) of type D or V. The space of type D (or V) is SD built on the base of the Dirak delta-function (correspondingly, on the base of the Hadamard finite part of integral). The equations under consideration can be solved explicitly only in rare cases. Therefore, the development of effective and theoretically based methods of their approximate solving in SD is an actual subject of mathematical analysis and computational mathematics. A number of results on this subject is obtained in works [6–9], where the direct special methods for solving of E3KFS (1) in a space of type D are proposed and substantiated. The first results on approximate solutions of E3KFS in certain SD X of type V are obtained in works [10, 11], where the authors develop polynomial for solving Eq. (1) in a space X.

In the present paper we use considerations and results of works [8–10], and propose a special version of the collocation method on the base of Hermite interpolation polynomials. This method is well-adapted for approximate solving Eq. (1) in class X. The main attention is paid to the substantiation of the method under consideration in the sense of book [12] (Chap. 1). Namely, we prove theorem on existence and uniqueness of solution to the corresponding equation, find bounds for errors of the approximate solution, and prove the convergence of sequence of approximate solutions to exact one in SD X. We consider also the questions of stability and conditionality of the approximating equations.

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