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Asymptotics of Spectrum of a Harmonic Oscillator Perturbed by a Nonsmooth Potential

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1. In this paper we study the operator $H = H^0 + V$ in $\mathbf{L}^2(\mathbf{R})$, where $H^0 = -d^2/dx^2 + x^2$, V is the operator of multiplication by a real, measurable, decreasing for $x \to \infty$ function. It is well-known (see, e.g., [1], p. 326) that the spectrum of the operator H^0 consists of the numbers $2n + 1$, and the corresponding normalized eigenfunctions are $\varphi_n(x) = H_n(x)e^{-x^2/2}/\sqrt{2^n n!\sqrt{\pi}}$, $n = 0, 1, 2, \ldots$, where $H_n(x)$ are the Chebyshev—Hermite polynomials. The asymptotics of eigenvalues of the perturbed operator $H = H^0 + V$ for smooth decreasing at infinity functions was first studied in detail in [2], where an etalon solution was obtained with the help of the Airy functions ([3], p. 377). Since $V(x)$ is not necessarily smooth, one cannot immediately apply the method of etalon solutions to the function $q(x) = x^2 + V(x)$. In this paper we use the apparatus of perturbation theory based on the study of the asymptotic representation of the kernel of the resolvent of the unperturbed operator.

Let λ_n stand for eigenvalues of the operator H^0, let P_n denote the corresponding projectors onto eigensubspaces, and let $R^0(\lambda)$ be the resolvent of the operator H^0, $R^0(\lambda) = (H^0 - \lambda)^{-1}$. According to [4], if V satisfies the condition

$$\lim_{n\to\infty} \sup_{|\lambda-\lambda_n|\le 1/2} \|R^0_n(\lambda)V\| = 0,$$

where $R^0_n(\lambda) = R^0(\lambda) - (\lambda_n - \lambda)^{-1}P_n$, then the spectrum of the operator $H = H^0 + V$ is defined with the help of the equation

$$\lambda = \lambda_n + (V\varphi_n, \varphi_n) - (VR_n(\lambda)V\varphi_n, \varphi_n). \tag{1}$$

Here $(\cdot,\cdot)$ is the scalar product in $\mathbf{L}^2(\mathbf{R})$, φ_n is the normalized eigenvector, corresponding to the eigenvalue λ_n,

$$R_n(\lambda) = \sum_{m=0}^{\infty} (-1)^m [R^0_n(\lambda)V]^m R^0_n(\lambda). \tag{2}$$

Let us represent the operator H^0 in the form

$$H^0 = H^0_D \oplus H^0_N,$$

where H^0_D and H^0_N are contractions of H^0 onto invariant subspaces, correspondingly, of odd and even functions from $\mathbf{H} = \mathbf{L}^2(\mathbf{R})$. Let $R^0(\lambda)$, $R^0_D(\lambda)$, and $R^0_N(\lambda)$ be resolvents of the operators H^0, H^0_D, and H^0_N. Since $\varphi_n(-x) = (-1)^n\varphi_n(x)$, we obtain

$$R^0_D(-x,t,\lambda) = -R^0_D(x,t,\lambda) = R^0_D(x,-t,\lambda),$$
$$R^0_N(-x,t,\lambda) = R^0_N(x,t,\lambda) = R^0_N(x,-t,\lambda)$$

for all $x, t \in \mathbf{R}$ and $\lambda \ne 2n + 1$.

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