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Solution of a Semicoercive Signorini Problem by a Method of Iterative Proximal Regularization of a Modified Lagrange Functional

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Abstract—Duality methods based on classical schemes for constructing Lagrange functionals are inapplicable for solving semicoercive variational inequalities in mechanics. In this paper we approximately solve a scalar semicoercive Signorini problem, using a duality method based on the iterative proximal regularization of a modified Lagrange functional. We realize the algorithm with the help of the finite element method on a sequence of domain triangulations.

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In solution methods for variational inequalities in mechanics based on finding saddle points of Lagrange functionals, the minimized functionals are assumed to be strongly convex. The convergence of such methods (in the direct variable) is provided only by the adjustment of the strong convexity constant with the step of the shift (with respect to the dual variable). Therefore algorithms for finding saddle points based on classical Lagrange functionals are inapplicable to semicoercive variational inequalities. In order to compensate this lack, in this paper we consider a modified analog of the Lagrange functional [1, 2].

1. The semicoercive Signorini problem.

In the variational definition the problem takes the form [3, 4]

$$\begin{gathered} J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 d\Omega - \int\limits_\Omega fv\, d\Omega \to \min, \\ v \in G = \{w \in W_2^1(\Omega) : \gamma w \geq 0 \text{ on } \Gamma\}. \end{gathered} \tag{1}$$

Here $\Omega \subset R^n (n = 2, 3)$ is a bounded domain with a sufficiently smooth boundary Γ, $f \in L_2(\Omega)$ is a given function, and $\gamma w \in W_2^{1/2}(\Gamma)$ is the trace of the function $w \in W_2^1(\Omega)$ on Γ.

Since the bilinear form $a(u, v) = \int\limits_\Omega \nabla u \nabla v\, d\Omega$ is not positive definite in $W_2^1(\Omega)$, problem (1) may have no solution. However if

$$\int_\Omega f d\Omega < 0, \tag{2}$$

then $J(v) \to +\infty$ as $\|v\|_{W_2^1(\Omega)} \to \infty$ ($v \in G$), and therefore the problem is resolvable [4]. Moreover, condition (2) provides the uniqueness of its solution. In what follows we assume this condition to be fulfilled.

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