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GENERALIZED SOLUTION OF MULTIDIMENSIONAL SEMILINEAR HYPERBOLIC SYSTEMS

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In the study of optimal control problems the discontinuity of controlling actions creates the necessity to consider solutions of differential equations, which define an admissible process, in a generalized sense. This problem is specifically sharp for systems of partial equations, where it is often impossible to construct a smooth and even a continuous solution which corresponds to the admissible control.

Various approaches can be put into the foundation of the notion of a generalized solution: Integral conservation laws, method of artificial viscosity, a way for passage to the limit in difference approximations, the apparatus of the theory of distributions (generalized functions), the concept of a potential of solution, and also other schemes (see [1], [2]). Unfortunately, not all of them are acceptable for one could use them in the theory of optimal control. For example, in the hyperbolic systems of multidimensional equations one of the most powerful notions of a generalized solution was introduced by Friedrichs (see [3]). He and his disciples proved the existence and uniqueness of a generalized solution which admits an arbitrarily accurate approximation with respect to the norm of the space L_2 by a sequence of smooth approximate solutions (see [3]–[9]). However, since the proof is carried out on the basis of integral operators of averaging, the estimate of the solution has the form of an energy inequality; therefore it cannot guarantee the smallness of the perturbation of solution at the points of the domain of independent variables under small perturbations of both the inhomogeneous part of the system and initial-boundary value conditions. Thus, from the principal point of view, the way to construct the generalized solution within the framework of this approach does not allow us both to substantiate the widely known in the theory of optimal control method of increments and even to catch the qualitative difference in the behavior of the increment of trajectory on different types of needle-shaped variations (see [10]).

In [11]–[16] they use a concept of generalized solution which was free of the above deficiencies. It was based on the following rather general idea from [1]: For a system of differential equations it is necessary to construct an equivalent integral system whose solution is announced as the generalized one. An intrinsic desire to extend this approach to hyperbolic systems of multidimensional equations for a long time was paralyzed in view of absence of the corresponding integral equivalent. In this article we suggest the solution of this problem on the basis of multidimensional analog of Riemann invariants.

1. Statement of problem

In the $m+1$-dimensional space of the independent variables (s,t), $s=(s_1,s_2,\dots,s_m)$ we consider the parallelepiped $\mathrm{P}=S\times T$, $S=S^{(1)}\times S^{(2)}\times\cdots\times S^{(m)}$, $S^{(j)}=[s_{0_j},s_{1_j}]$, $j=1,2,\dots,m$, $T=[t_0,t_1]$. Let Ω, G, $\mathcal{D}_0$, and $\mathcal{D}_1$ be its complete, lateral, lower, and upper surfaces, respectively,