# PRACTICAL WORK. MEDICAL PHYSICS

For English-speaking students of medical, biomedical, and pharmaceutical fields of study

### KAZAN FEDERAL UNIVERSITY INSTITUTE OF PHYSICS DEPARTMENT OF MEDICAL PHYSICS

# PRACTICAL WORK. MEDICAL PHYSICS

For English-speaking students of medical, biomedical, and pharmaceutical fields of study

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Manual of laboratory work "Practical work. Medical physics" is intended for students of foreign groups of medical specialties of the university. For each work, a brief overview, installation manuals, step-by-step instructions, and a list of questions for self-study are provided.

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#### Introduction

The branch of Kazan Federal University in Cairo opened a year ago. It is necessary to fine-tune the educational process according to the standards of our university, but taking into account the specifics of first-year students from Africa and the Middle East. Right-to-left writing, variety of systems of numerals and units adopted in different countries of this region, and even different systems of mathematical notation, different students` levels, wide spectrum of educational curriculums result in substantial challenges that students and teachers have to get over together. In addition, the need for this manual is due to the set of lab equipment purchased for the Medical Physics course.

Practical work with a real experiments in a physical laboratory is an integral part of medical education. We hope that this manual will be useful as student as assistants of teachers in the KFU branch in Cairo.

#### Acknowledgments

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R.A. Freedman. – Pearson Education, Inc.; 15th edition, 2019. – 1600 p. Serway, R.A. Physics for Scientists and Engineers with Modern Phys-

ics / R.A. Serway, J.W. Jewett, Jr. – Cengage Learning, 2018. –1484 p. URL: https://www.ld-didactic.com

### Safety precautions

1. Students who have been instructed in safe working practices and signed the safety logbook will be allowed to participate in lab class.

2. Students must study the description of the work to be carried out before the laboratory session.

3. Each student <u>MUST</u> follow the rules of Physics Lab:

a) Please exercise caution when dealing with electrical devices. Do not allow the use of faulty electrical plugs, sockets or exposed wires to connect electrical equipment;

b) Never touch electrical equipment with wet hands;

c) Do not set equipment too close to the edge of the table;

d) Use equipment and tools for their intended purpose;

e) Check all apparatus for defects before performing any experiments. Do not use damaged, cracked or otherwise defective glassware;

f) If you break a device containing mercury or find any defect in it, report it to your teacher or engineer immediately. Mercury vapor is a strong poison;

g) Coats, bags, and other personal items should be stored in the proper areas; not on tables, not on bench tops, not in the aisle ways. Student, teacher, and engineer should be able to easily move around the lab room;

h) Do not eat or drink in the lab. Do not use lab containers and utensils for food and drinks storage.

4. At the end of the work, switch off the equipment and tidy up the work area.

5

#### Drafting of the report

The laboratory report consists of title, objective and tasks of the work, theory, experimental setup, algorithm of measurements, analysis of results (tables, plots, calculations), conclusion.

"Title" of the work should be taken from the description of this lab. The "objectives" should be briefly stated in the description provided for the lab work. "Theory" should contain only that information that is directly related to the lab work. Definitions of basic quantities and basic equations which will be used for calculations must be given. Be sure to make drawings and explanatory drawings. It is necessary to list the equipment and measuring devises in the "Experimental setup" section. The "Algorithm of measurements" is a list of the steps followed to carry out the experiment. This section should contain a description of the main stages of the measurement of quantities, i.e. the experimental methodology. The experimental results are presented in the form of *tables*. The "analysis of results" provides the necessary calculations using the equations that were given in the "Theory". It is also necessary to build *plots* and tables according to this manual. "Conclusions" are formulated according to the objectives. If problems arose during the work either with the theoretical model or with the experimental methodology, then you should definitely point this out, not forgetting to consider the reasons for the occurrence of these difficulties. This section compares theoretical calculations with experimental results. If there are inconsistencies, the reasons for their occurrence are analysed.

#### Units of physical quantities

The International System of Units (abbreviation SI, from French Système international d'unités) is used in science and industry. It is based on seven basic units: meter, kilogram, second, ampere, kelvin, mole, candela, and two additional ones: radians and steradians.

<u>Meter</u> (m) is the length of the path light travels in a vacuum in (1/299792458) s (seconds).

<u>*Kilogram*</u> (kg) is a mass equal to the mass of the international prototype of the kilogram (a platinum-iridium cylinder with a height and diameter

of 39 mm; this is the mass of 1 liter of water at a temperature of 277 Kelvin), stored at the International Bureau of Weights and Measures in Sèvres, near Paris.

<u>Second</u> (s) is a time equal to 9192631770 periods of radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom.

<u>Ampere</u> (A) is the strength of a constant current, which, when passing through two parallel straight conductors of infinite length and negligible cross-section, located in a vacuum at a distance of 1 m from each other, creates a force between these conductors equal to  $2 \times 10^{-7}$  N for each meter of length.

<u>*Kelvin*</u> (K) is 1/273.16 part of the thermodynamic temperature of the triple point of water.

<u>Mole</u> (mol) is the amount of substance in a system containing the same number of structural elements as there are atoms contained in a  $^{12}$ C nuclide weighing 0.012 kg.

<u>*Candela*</u> (cd) is the luminous intensity in a given direction of a source emitting monochromatic radiation with a frequency of  $540 \times 10^{12}$  Hz, the energy intensity of light in this direction is 1/683 W/sr (Watt/Steradian).

<u>*Radian*</u> (rad) is the angle between two radii of a circle, the length of the arc between which is equal to the radius.

<u>Steradian</u> (sr) is a solid angle with a vertex at the center of the sphere, cutting out an area on the surface of the sphere equal to the area of a square with a side equal to the radius of the sphere.

The remaining units, called derivatives, are derived from physical laws that connect them with the above-mentioned basic units.

International System of units
-------------------------------

SI Base Units				
Base Quantity	Name	Symbol		
Length	meter	m		
Mass	kilogram	kg		
Time	second	S		
Electric current	ampere	А		
Thermodynamic	kelvin	K		
temperature				
Amount of substance	mole	mol		
Luminous intensity	candela	cd		

SI Derived Units				
<b>Base Quantity</b>	Name	Symbol	Equivalent SI units	
Frequency	hertz	Hz	s <sup>-1</sup>	
Force	newton	Ν	$m \cdot kg/s^2$	
Pressure	pascal	Pa	N/m <sup>2</sup>	
Energy	joule	J	N∙m	
Power	watt	W	J/s	
Electric charge	coulomb	C	s·A	
Electric potential	volt	V	W/A	
Electric resistance	ohm	Ω	V/A	

SI Prefixes				
Factor	Name	Symbol	Numerical Value	
$10^{12}$	tera	Т	1 000 000 000 000	
$10^{9}$	giga	G	1 000 000 000	
$10^{6}$	mega	М	1 000 000	
$10^{3}$	kilo	k	1 000	
$10^{2}$	hecto	h	100	
$10^{1}$	deka	da	10	
10-1	deci	d	0.1	
10-2	centi	С	0.01	
10-3	milli	m	0.001	
10-6	micro	μ	0.000 001	
10-9	nano	n	0.000 000 001	
10-12	pico	р	0.000 000 000 001	

#### **Plots**

A plot is necessary for a clear presentation of the results (the relationship between the quantities of interest), so the main requirement for them is accurate and clear execution. Plots are made on graph (millimeter) paper. Scales are specified along the vertical and horizontal axes. The scale along the axes is chosen so that the experimental points are located over the entire area of the plot. The plots should be easy to read, for this it is necessary that the plot cell corresponds to 1, 2, 5, 10, 0.1, 0.01, 0.001, etc. The reference point does not have to start from zero; sometimes it is more convenient to choose a rounded number other than zero, and thus increase the scale. The coordinate axes are labelled with letters indicating the quantities to be fixed and their dimensions in SI. An example of building a plot based on experimental data is shown in Fig. 1. The obtained experimental data are points (small circles).



Fig. 1. Example of the plot

Experimental points do not fall exactly on the curve (straight line) of the dependence, but are grouped around it randomly due to measurement errors. Points should not be connected by straight segments, resulting in a kind of broken line. It is necessary to draw smooth curves corresponding to the physical dependencies being studied. First you need to find out what kind of dependence there is (linear, exponential, etc.).

Then an averaging curve is drawn across the points. Usually experimental points do not lie on it, but have some scatter (dispersion). The curve is drawn so that the experimental points deviate uniformly from it (Fig. 1). If several curves are plotted, then it is possible to draw them by different colors or show them by different lines (solid, dotted, dash-dotted...) and sign the title.

Summarize, components of a plot:

<u>Title</u>. The title of the plot is written on the top or bottom of the plot. The title should be clear, descriptive, and self-explanatory.

<u>Axes</u>. Take the independent variable as the abscissa (x-axis) and the dependent variable as the ordinate (y-axis) (unless you are instructed otherwise).

<u>Labels</u>. Label each axis to identify the variable being plotted and the units (in SI) being used.

<u>Scale</u>. Choose a convenient scale for each axis so that the plotted points will occupy a substantial part of the plot, do not choose a scale which is difficult to plot and read.

<u>Fitting</u>. If the experimental data is theoretically a straight line, draw the best straight line through the points with a straight edge. The line should be drawn with about as many points above it as below it.

<u>The slope</u>. In plotting linear relationships, you frequently will be asked to find the slope of the line you have fitted to the data <u>(this slope usually has a physical meaning)</u>.



$$slope = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Fig. 2. Example of the slope

### **1. MECHANICS**

### 11. Caliper gauge with Vernier

#### *Objective*

Studying the measurements by a caliper.

#### Tasks

Determination of an outside, an inside, and a depth dimension by means of a caliper gauge.

Improvement of the measuring accuracy by means of a vernier.

### <u>Vernier</u>

Vernier is the title given to the auxiliary scale of measuring instruments, which serves for counting divisions fractions of the main scale. Vernier allows to increase the accuracy of measurements by 10-20 times.

Most instruments use linear or angular (circular) scales. The reading on the device is a measurement of the lengths of straight or arc segments. In the case when the relative accuracy of measuring length is such that one can be satisfied with absolute accuracy of hundredths or even tenths of a millimeter, and for angles minutes or fractions of minutes.



Fig. 11.1. a) zero-position; b) setting 10.8 mm

The simplest is the decimal vernier, which makes it possible to measure length with an accuracy of 0.1 divisions of the main scale. This vernier is an additional ruler divided into segments. The length of the entire vernier is equal to nine whole scale divisions. Thus, if the length of one division of the vernier is X, and the length of one division of the scale is Y = 1 mm, then  $10 \cdot X = 9$  mm. Therefore, the length of each vernier division will be 0.9 mm. If the zero stroke of the vernier, and, consequently, the tenth, exactly coincides with any scale stroke, then all other vernier strokes do not coincide with the scale strokes (Fig. 11.1a). If the zero stroke of the vernier does not coincide with the scale stroke, then there is a stroke that coincides with some scale stroke much better (Fig. 11.1b).

The smallest value that can be measured using a vernier is determined by the difference  $\Delta X = Y - X$  between the length of the scale and the length of the vernier division. This difference will be in our case the division price or the accuracy of the vernier: at X = 0.9 mm, Y = 1 mm,  $\Delta X = 0.1$  mm.

#### How to use a vernier?

When taking a reading, it is necessary to determine the distance L between the zeros of the vernier and the main scale.

In Fig. 11.1b, this distance consists of 10 scale divisions "traversed" by the vernier zero, that is, 10 mm and a segment  $\Delta L$ , the length of which is equal to the distance from the tenth scale line to the vernier zero with an accuracy of 0.1 mm.

As can be seen from Fig. 11.1b, the eighth stroke of the vernier, marked with an arrow, exactly coincides with the scale stroke. The seventh stroke does not coincide with the scale stroke to the extent that the length of the vernier division is shorter than the length of the scale division, that is, by 0.1 mm. The sixth line of the vernier does not coincide with the scale line by 0.2 mm, since the length of two vernier divisions is 0.2 mm shorter than the length of two scale divisions. The zero line of the vernier does not coincide with the scale line by 0.8 mm, since eight divisions of the vernier are shorter than eight divisions of the scale by 0.8 mm. The distance between the zero line of the vernier and the tenth line of the scale is exactly equal to the segment  $\Delta L$ . Thus, the segment  $\Delta L$  is equal to 0.8 mm. In other words, to find tenths of a scale division using a decimal vernier, you need to multiply the number of the "matching" vernier division by 0.1, that is, by the price of the vernier division. So, the final measurement result for Fig. 11.1b is 10 mm + 0.8 mm = 10.8 mm.

### <u>Caliper</u>

Measuring length is one of the oldest measurement problems. Therefore, simple methods of measuring length are well known. For small lengths, for example, the caliper gauge is often used (Fig. 11.2). A caliper is a device used to measure linear dimensions with an accuracy of 0.1 to 0.02 mm.



Fig. 11.2. Caliper gauge with a vernier: a – is an acute measuring shank;
b – is a long measuring shank; c is a slide with vernier; d – is a ruler with a millimeter scale; e – is a feeler for depth measurements

It consists of a ruler with millimeter scale, to which the measuring shank 1 has been attached at a right angle. The ruler simultaneously serves as a guide for the caliper gauge, which carries the measuring shank 2. The zero-mark of the gauge coincides with the zero-mark of the ruler if both measuring shanks are in contact.

There are two types of measuring shanks 1 and 2 available. Long measuring shanks serve to determine outside dimensions, while acute measuring shanks measure inside dimensions.

By means of an additional feeler, depth measurements can be performed (Fig. 11.2).

Various dimensions x of a workpiece are being determined several times during the test. Apart from the average value  $(\bar{x})$  of the sample size also the standard deviation  $(S_x)$  is calculated.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i, \qquad (11.1)$$

where n is number of individual measurements.

$$S_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}.$$
 (11.2)

The latter is a measure of the spread of the individual measurements around the average value. It is being compared to the read-out accuracy of the caliper gauge.

#### *How to use a caliper?*

Step 1. Look at main scale

When taking a measurement, you should first read the value on the main scale. The smallest value that can read from the main scale is 1mm (indicated by a single increment). The value on the main scale is the number immediately to the left of the 0 marker of the vernier scale. It is 13 mm in the example in Fig. 11.3.



Fig. 11.3. Caliper gauge with a Vernier

Step 2. Look at vernier scale

The vernier scale of a metric caliper has a measuring range of 1 mm. In the example we will be looking at, the vernier scale is graduated in 50 increments. Each increment represents 0.02 mm. It is 0.42 mm in Fig. 11.3. However, some vernier scales are graduated in 20 increments, with each one representing 0.05 mm. Some vernier scales are graduated in 10 increments, with each one representing 0.1 mm.

Step 3. Add both values together

Identify the increment that lines up most accurately with an increment on the main scale when reading the vernier scale. This value will make up the second part of your measurement. To get your total reading, add both the value from the main scale and the value from the vernier scale together. It is 13 mm + 0.42 mm, the reading is 13.42 mm in the example in Fig. 11.3.

### Experimental setup

- Precision caliper gauge;
- Workpiece with outside, inside and depth dimensions.

### Algorithm of measurements

Task 1. Determination of an outside dimension

- 1. Loosen the catch of the gauge, bring the object to be measured between the long measuring shanks and push the measuring shanks into close contact without tilting.
- 2. Estimate the outside dimension A (Fig. 11.4) on the millimeter scale and follow up with a more precise vernier reading.
- 3. Remove caliper gauge, reset, and repeat measurement 5 times.



Fig. 11.4. Determination of an outside dimension with the long measuring shanks

4. Fill in the Table 11.1 with results of your measurements.

*Table 11.1* 

Measurement	Millimeter scale	Vernier (mm)	Result (m)
number ( <i>i</i> )	(main scale)		
	(mm)		
1			
2			
3			
4			
5			

Results of the outside measurement

Average value: \_\_\_\_\_m

### Task 2. Determination of an inside dimension

- 1. Loosen the catch of the gauge. Guide the object to be measured over the acute measuring shanks and slide the measuring shanks apart without tilting.
- 2. Estimate the inside dimension B (Fig. 11.5) on the millimeter scale and follow up with a more precise vernier reading.
- 3. Remove caliper gauge, reset, and repeat measurement 5 times.
- 4. Fill in the Table 11.2 with results of your measurements.

Table 11.2

Measurement	Millimeter scale	Vernier (mm)	Result (m)
number (i)	(main scale)		
	(mm)		
1			
2			
3			
4			
5			

### Results of the inside measurement

Average value: \_\_\_\_\_m



Fig. 11.5. Determination of an inside dimension with the acute measuring shanks

### Task 3. Determination of a depth

- 1. Loosen the catch of the gauge. Set the ruler on the edge of the hole and by moving the slide lower the feeler until it makes contact with the base.
- 2. Estimate the depth C (Fig. 11.6) on the millimeter scale and follow up with a more precise vernier reading.
- 3. Remove caliper gauge, reset, and repeat measurement 5 times.
- 4. Fill in the Table 11.3 with results of your measurements.



Fig.e 11.6. Performing a depth measurement by means of a feeler

Table 11.3

### Results of the depth measurement

Measurement number (i)	Millimeter scale (main scale) (mm)	Vernier (mm)	Result (m)
1			
2			
3			
4			
5			

Average value: \_\_\_\_\_m

<u>Analyse the results.</u> Make <u>conclusions</u> corresponding to the objective of the laboratory work.

### Questions

- 1. What is the purpose of using calipers?
- 2. How many scales are included in calipers?
- 3. The working principle of calipers. How to read a caliper?
- 4. How is the operational functionality of a caliper checked?
- 5. How to measure internal dimensions using a caliper?

#### 12. Micrometer

#### **Objective**

Studying the measurements by a micrometer.

Task

Measuring the diameters of thin wires and examining the accuracy of measurement.

Small thicknesses are often measured by means of a micrometer. A micrometer screw consists of a massive bow carrying a rigid measuring jaw on the left and a travelling measuring jaw on the right (Fig. 12.1). The measuring jaws are opened or closed, respectively, by turning a thimble around a cylinder, which is rigidly connected with the bow.



Fig. 12.1. Micrometer:  $\mathbf{a}$  – is a rigid measuring jaw,  $\mathbf{b}$  – is a travelling measuring jaw,  $\mathbf{c}$  – is a cylinder with rough scale,  $\mathbf{d}$  – is a thimble with fine scale,  $\mathbf{e}$  – is a screw with friction clutch,  $\mathbf{f}$  – is a bow

A scale on the cylinder corresponds to the distance between the measuring jaws in steps of 0.5 mm. The zero of the scale is reached when the measuring jaws are completely closed. When the thimble is turned by a full revolution, the right measuring jaw is moved by half a millimeter. The accuracy of measurement is enhanced by an additional scale engraved around the thimble, having 50 graduation marks that correspond to a change of the distance between the measuring jaws of 10  $\mu$ m. Thus, the accuracy of reading is approx. 2  $\mu$ m (Fig. 12.1). The object to be measured is clamped between the measuring jaws. In order to prevent the object from being deformed, a screw is turned which is connected to the thimble via a friction clutch.

In the experiment, the thicknesses (d) of different wires are each measured several times. Apart from the average value ( $\bar{d}$ ) of the wire diameter

$$\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i, \qquad (12.1)$$

where *n* is number of individual measurements. The standard deviation  $(S_d)$  is calculated also.

$$S_d = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (d_i - \bar{d})^2}$$
(12.2)

The latter is a measure of the spread of the individual measurements around the average value. It is compared with the accuracy of reading of the micrometer.

Next, the meaning of the friction clutch is investigated by measuring a soft wire with and without using the friction clutch.

The action of the micrometer is based on the property of the screw, when turning it, to make a translational movement proportional to the angle of rotation. When measuring, the object is clamped between the heel and the micrometer screw.

To rotate the drum, only the friction head is used. After the maximum degree of pressure on the object is reached (500-600 g), the friction head begins to slip, emitting a characteristic crackling sound. Thanks to this, the clamped object is deformed relatively little (its dimensions are not distorted).

Before you start working with the micrometer, you should make sure that it is in good working order. To do this, by rotating the friction head, the micrometer screw is brought into contact with the heel. The moment of contact is determined by the ratchet signal. In this case, the edge of the drum should be located above the zero division of the main scale, and the zero of the drum should be located against the line on the tube. If these conditions are not met, then in all further measurements the systematic error of the micrometer should be taken into account, equal to the number of drum divisions that corresponds to the closed micrometer screw and heel. If this deviation is large, then the micrometer needs adjustment. Rotating the screw with force (by the drum) after the ratchet has started is prohibited, as this will lead to damage to the device.



Fig. 12.2. Representation of a distance (*d*) on the rough scale (**c**) and on the fine scale (**d**): d = 0.5 mm + 0.150 mm = 0.650 mm

#### Measurement Features

When the drum scale reading is slightly less than 50 (or 100), the next division of the drum scale is usually shown from under the edge of the drum. This is especially true when measuring with micrometers having scales with half divisions. Such a scale is shown in Fig. 12.2. The upper row of divisions on the main scale marks half fractions of the main scale. Naturally, in this case, the drum scale has half as many divisions as the one shown in Fig. 12.2.



Fig. 12.3. Representation of a distance (d) on the rough scale (c) and on the fine scale (d): a) d = 8.0 mm + 0.48 mm = 8.48 mm;b) d = 5.0 mm + 0.5 mm + 0.47 mm = 5.97 mm

The last visible lower division in Fig. 12.3 a corresponds to 8 mm and, in addition, a further upper division is shown. The question arises as to the number of whole and half divisions should be counted correctly: 8.0 mm

or 8.5 mm? In this case, the upper division that appeared should not be taken into account, because the drum scale reading of 48 divisions indicates that the drum edge has moved away from the last lower visible eighth division 0.48 mm; therefore. in this case the will be by count 8.0 mm + 0.48 mm = 8.48 mm. If the edge of the drum had moved 0.48 mm away from the top division, there would be a noticeable gap of nearly half a millimeter between that division and the drum.

Fig. 12.3b shows the position of the drum at which the sixth division of the main scale is already visible from under its edge. However, the drum reading is 0.47 mm. This means that up to the sixth whole millimeter, it is necessary to turn the drum by three divisions of its scale (to move the drum edge by 0.03 mm). Thus, in this case, the sixth division of the main scale should be disregarded and, therefore, the reading will be 5.0 mm + 0.5 mm + 0.47 mm = 5.97 mm.

### Experimental setup

- Precision micrometer screw gauge;
- Copper wire;
- Brass wire.

### Algorithm of measurements

Task 1. Measuring the thickness of a wire at several positions

- 1. Take the brass wire between the measuring jaws.
- 2. Bring the measuring jaws together by turning the screw until the wire is clamped and the thickness to be read no longer changes.
- 3. Read the thickness (*d*).
- 4. Clamp the brass wire at another four positions, each time reading the thickness d.
- 5. Repeat the measurements with the copper wire.
- 6. Fill in the Tables 12.1 and 12.2 with results of your measurements.

*Table 12.1* 

Thickness of the b	rass wire
--------------------	-----------

Measurement number (i)	Rough scale	Fine scale	Result (m)
1			
•••			
5			

Average value: \_\_\_\_\_ m

### Thickness of the copper wire

Measurement number (i)	Rough scale	Fine scale	Result (m)
1			
•••			
5			

Average value: \_\_\_\_\_ m

Task 2. Demonstrating the deformation when the friction clutch is not used

- 1. Take the brass wire between the measuring jaws and bring the measuring jaws together by turning the screw until the wire is clamped.
- 2. Continue turning the screw and read the thickness of the wire several times.
- 3. Then turn the thimble, and read the thickness of the wire several times:

after the wire has been clamped gently:  $d = \__m mm = \__m m$ after turning the screw repeatedly:  $d = \__m mm = \__m m$ after turning the thimble repeatedly:  $d = \__m mm = \__m m$ 

<u>Analyse the results</u>. Make <u>conclusions</u> corresponding to the objective of the laboratory work.

### Questions

- 1. What is the purpose of using micrometers?
- 2. What are the component parts of a micrometer? How many scales are included in micrometers?
- 3. The working principle of micrometers. How to read a micrometer?
- 4. How is the operational functionality of micrometers checked?
- 5. What is the division value of a micrometer?

### 13. Determining the volume and density of solids

#### **Objective**

Determining the density of the bodies.

**Tasks** 

Measuring the volumes of different bodies according to the overflow method.

Comparing the volumes measured with those calculated from the dimensions.

Calculate the density of different bodies.

The density  $\rho$  of a homogeneous substance is usually determined by measuring the mass *m* and the volume *V* separately and then calculating the density from these quantities:

$$m = \rho \cdot V. \tag{13.1}$$

The volumes of the bodies are determined by the volume of the liquid that the bodies displace from an overflow reservoir. This procedure is tested in the experiment with regular bodies whose volumes can be calculated from their linear dimensions.

The laboratory scales used in this lab are lever scales. The moment of weight of the bowl with the object under study is balanced by the moment of weight of the weights. The latter changes by moving weights along several rails. Opposite the fixed positions of the weights, the values of the balanced mass are engraved.

The accuracy of the scale is 0.01 g, the measurement limit is 311 g.

The scales are equipped with a locking device. It secures the balance beam when not in use and protects the prism edge from wear. Typically, the scales must be locked (the rocker arm is secured). When weighing, the rocker arm is released by turning the foot near the base of the scale.

Before weighing, make sure that the scales are correctly installed and that they are in equilibrium when unloaded. If necessary, with the extreme left positions of the weights, it is necessary to achieve balance by moving the weight along the screw near the point of attachment of the bowl to the rocker arm.

To determine whether the scale is in balance, there is no need to wait for it to stop. The scales are balanced when the needle deviates from the balanced position by the same number of divisions when oscillating.

### Weighing Rules

A body may be place on or remove from the pan only when the scale is locked.

The first to move is the weight, which, in the opinion of the weigher, will most likely overwhelm the body. If the weight is too heavy, the weight is moved one division to the left until the body begins to pull on the weights. If the body is too heavy, then the weight is moved to the right until the weight outweight the body, and then it is shifted one division to the left.

Repeat these steps for the smaller weight.

### Experimental setup

- Glass cylinder;
- Graduated cylinder;
- Caliper;
- Balance;
- Cubes and balls.

### Algorithm of measurements

- 1. Fill in the graduated cylinder with water. Record the occupied volume value based on the liquid level in the cylinder.
- 2. Place the cube to be measured completely in the water in the glass cylinder. Determine the difference in liquid volume using a graduated cylinder. Record the measurement results.
- 3. Take the cube out of the water, measure its dimensions with a caliper, and write them down.
- 4. Determine the mass of the cube using a balance, and record the measurement results.
- 5. Repeat the measurements with other objects given by the teacher.
- 6. Finally, determine the volume and mass of an arbitrary object.
- 7. Fill in Table 13.1, the volumes measured according to the overflow method are compared with those calculated from the dimensions using a caliper. Calculate the density of the studied body using the Eq. (13.1).
- 8. Compare the resulting density value with the tabular value of this material.

*Table 13.1* 

Body	Measured	Calculated	Calculated densi-	Equation
	Volume	Volume	ties	
	$V(m^3)$	$V(m^3)$	$\rho (kg/m^3)$	
Cube				$V = d^3$
Ball				$(d)^{3}$
				$V = \frac{1}{3} \cdot \pi \cdot \left(\frac{1}{2}\right)$
Cylinder				$(d)^2$
				$V = \pi \cdot \left(\frac{1}{2}\right) \cdot h$

Measured and calculated volumes of the test bodies

<u>Analyse the results.</u> Make <u>conclusions</u> corresponding to the objective of the laboratory work.

### Questions

- 1. What is the difference between vector and scalar quantities (give examples of vector and scalar quantities in physics)?
- 2. What are mass, density, and volume? Definition. Units of measurement in SI.
- 3. What is a rigid body?
- 4. What equation is used to calculate mass?
- 5. What does the density of a body depend on?
- 6. The working principle of a caliper.

### 14. Simple pendulum

#### **Objective**

Studying the acceleration of free fall (gravity acceleration).

#### <u>Tasks</u>

Studying the method of measuring of the acceleration of free fall with the help of a simple pendulum.

Estimation of possibility of describing the given real pendulum by the model of a simple pendulum.

Acceleration of free fall  $\vec{g}$  is the acceleration with respect to the Earth at which a released body begins to fall down. This acceleration is defined by the sum of the force of gravity (attraction to the Earth) and the centrifugal force of inertia.

A simple (mathematical) pendulum is an imaginary pendulum with all its mass located at one point, while the distance l from this point to the centre of suspension (pivot) being constant during oscillations. Simple calculations show that at small angles of deviation from the vertical, the period of oscillations of the pendulum is:

$$T = 2\pi \sqrt{\frac{l}{g}}.$$
 (14.1)

Now, the idea of one possible approach to determining the acceleration of free fall is clear: it is necessary to measure the length and period of a simple pendulum.

However, a question arises whether properties of a real pendulum are properly described by the model of a simple pendulum?

Note that Eq. (14.1) shows that the period of oscillations of the simple pendulum is proportional to  $l^{0.5}$ . If this correlation is true for a given real pendulum, it can be considered a simple pendulum, and the acceleration of free fall will be defined from the equation:

$$g = \frac{4\pi^2 l}{T^2}.$$
 (14.2)

### Experimental setup

- Massive ball on inextensible rope;
- Ruler;
- Timer.

Algorithm of measurements

- 1. Shorten the rope so that its length is 10-15 cm.
- 2. Measure the length of the pendulum *l*, which is the distance between the centre of suspension (pivot) and the centre of the ball.
- 3. Deflect the ball so that the angle between the rope and the vertical does not exceed 10°, and release the ball.
- 4. Measure the duration of 10 full oscillations  $t_{10}$  and find the period  $T = t_{10}/10$ .
- 5. Add 5–10 cm to the length of the pendulum (use a roller at the attachment point).
- 6. Repeat steps 2–4.
- 7. Repeat steps 5–6 until the pendulum is over 100 cm long.
- 8. Analyse the results.
- 9. Record into the Table 14.1 results of your measurements and calculations.

*Table 14.1* 

Results of measurements and calculations

N⁰	<i>l</i> (cm)	$t_{10}$ (s)	<i>T</i> (s)	$T^2$ (s <sup>2</sup> )
1				
2				

Build a plot  $T^2$  vs. *l*. Do a linear approximation, find slope (acceleration of free fall) and intercept, Eq. (14.2). Carry out a standard statistical analysis of this plot: calculate average value  $\langle g \rangle$ , dispersion, and error of the acceleration of free fall determination.

<u>Analyse the results</u>. Make <u>conclusions</u> corresponding to the objective of the laboratory work.

### Questions

- 1. Forces of inertia.
- 2. Equations of motion of a material point relative to a rotating frame (the Earth).
- 3. The reasons for the dependency of the acceleration of free fall on the position on the Earth's surface.

- 4. Vector and vector's components; vector coordinates; projection of a vector onto a given direction.
- 5. Angular velocity and angular acceleration.
- 6. Inertial and non-inertial coordinate system.
- 7. Moment of inertia.
- 8. Centre of masses.
- 9. Acceleration of free fall.
- 10. Simple harmonic oscillations.

### 15. Torsion pendulum

#### **Objective**

Studying the damped oscillations of a torsion pendulum.

Tasks

Measuring the amplitude of rotational oscillations as function of time.

Determination of the damping coefficient of a torsion pendulum.

The rotary oscillations are a special case among various mechanical oscillator models (physical pendulum, spring pendulum etc.) which allow investigating the most important phenomena which occur in all types of oscillations. A simple harmonic oscillator is a system that performs sinusoidal (or cosinusoidal) oscillations about an equilibrium point with constant amplitude ( $A_0$ ) and constant angular frequency (angular speed) ( $\omega$ ), the SI unit is radian per second (rad/s). The equation of motion is  $A = A_0 \cdot \sin(\omega \cdot t + \varphi_0)$ , A is a parameter of the system (coordinate, density, etc.), t is time, and  $\varphi_0$  is the initial phase.

Amplitude can be defined as the maximum deviation of a system from the point of equilibrium. In the context of a rotary system, it is the maximum angle. A period (the SI unit is second) is defined as the time required for one full oscillation. Frequency is the number of oscillations per second (the SI unit is hertz (Hz)). Period ( $T_0$ ) and angular frequency are dependent from each other by the following equation:

$$\omega_0 = \frac{2\pi}{T_0}.\tag{15.1}$$

### Damping oscillations

Owing to unavoidable frictional forces (torques) the amplitude of oscillations inevitably decreases with time. The movement of a free, damped (rotary) oscillating system (the rotor) with damping can be classified into two distinct cases.

1) In case of underdamped oscillator ( $\gamma < \omega_0$ ) The movement can be described by the equation,

$$\phi(t) = \phi_0 \, e^{-\gamma t} \cos(\omega_1 t), \tag{15.2}$$

where  $\phi_0$  is initial angel of rotation at initial time t = 0,  $\gamma$  is damping constant,  $\omega_1$  is the frequency of damped oscillations and is determined as

 $\omega_1 = \sqrt{\omega_0^2 - \gamma^2}$ ,  $\omega_0$  is the characteristic angular frequency of an "undamped" system.

The amplitude decreases by the factor  $e^{-\gamma t}$ , the damping constant can be determined by:

$$\gamma = \frac{1}{T} \ln(\frac{\Phi_n}{\Phi_{n+1}}), \qquad (15.3)$$

where,  $\phi_n$  and  $\phi_{n+1}$  is two successive amplitudes.

2) In case of overdamped oscillator ( $\gamma > \omega_0$ )

$$\phi(t) = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}. \qquad (15.4)$$

The oscillating system approaches the equilibrium position asymptotically at one oscillation.

#### Driven oscillations

When the periodic torque is applied by the exciter with a frequency  $\omega_{ext}$  that has the form:

$$\phi(t) = \phi_0 \sin(\omega_{ext} t + \delta), \qquad (15.5)$$

which means that after a while, when free oscillations have damped, the rotor oscillates stationary with the frequency of the exciter  $\omega_{ext}$ . Thus, the amplitude of forced oscillations is:

$$\phi_0 = \frac{\omega_0^2 \theta_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2 \omega^2}}.$$
 (15.6)

The amplitude has a maximum at resonance frequency  $\omega_R$ . To be maximal, the radicand in the denominator should be minimal.

$$(\omega_0^2 - \omega^2)^2 + 4\gamma^2 \omega^2 = 0$$
,  $\omega_R^2 = \sqrt{\omega_0^2 - \gamma^2}$ . (15.6)

The lower the damping, the less the resonance frequency differs from the natural frequency  $\omega_0$  and the larger is the amplitude. In the limit of disappearing damping, the amplitude at the resonance frequency  $(\omega_{ext} = \omega_0)$  would tend towards infinity (so-called resonance catastrophe).

The amplitude of the forced oscillations tends towards zero for very high frequencies ( $\omega_{ext} \gg \omega_0$ ). For very low frequencies ( $\omega_{ext} \ll \omega_0$ ) the amplitude tends towards the value  $\omega_0^2 \theta_0$ . That is why the resonance curve is not symmetrical with respect to the resonance frequency $\omega_R$ .



Fig. 15.1. Schematic representation of various damped oscillation curves:a) underdamped oscillator; b) overdamped oscillator

### Experimental setup

• torsion pendulum (1) with an electromagnet in the form of a coil (5) and a motor that creates a driving oscillating force (Fig. 15.2);

• direct current source of the electromagnetic coil of the torsion pendulum (2);

- power cord (3);
- direct current source for the pendulum motor (4);
- personal Computer (PC) connected with CassyLab device;
- red and blue cables (100 cm).

#### Safety notes

! The current through the eddy current brake should not exceed 2 A for a long time.

A torsion pendulum can be used to study free and forced rotational harmonic oscillations. An electromagnetic coil with current (5) slows down (dampens) these oscillations. The greater the current passing through the coil, the greater the braking effect. In addition, the torsion pendulum can be excited by an oscillating force through an eccentric rod (6) (Fig. 15.2) controlled by a motor. The speed of the eccentric thrust of the motor can be changed using two knobs: coarse and fine adjustment (4).

#### Algorithm of measurements

1. Create the Table 15.1 in a notebook, to record the results of the experiment and calculations.

*Table 15.1* 

I(A)	$\phi_n(rad)$	$\phi_{n+1}(rad)$	<i>t</i> <sub>1</sub> (s)	<i>t</i> <sub>2</sub> (s)	T(s)	γ

### Results of measurements and calculations



Fig. 15.2. Experimental setup

- 2. Switch on the Mobile-CASSY 2. Turn the PC on.
- 3. Check that the torsion pendulum and PC is connected to the Mobile-CASSY 2.
- 4. Open the CASSY's program on the PC. Click on connect bottom to connect to the Mobile-CASSY. Then this window will appear, select "Show Measuring Parameters" (Fig. 15.3).



Fig. 15.3. Connecting between a personal computer and the CASSY device

- 5. Set the parameters for the experiment as shown in Fig. 15.4. (left).
- 6. By rotating the V knob on the current source (2), set the current supplied to the damping coil electromagnet to 0 A.
- 7. Move the pointer of the pendulum to the limit position and let the pendulum swing until it comes to rest and at the same time start recording graph on the PC by clicking the bottom "**Measuring time**" as shown in Fig. 15.4. (right).
- 8. When the motion has stopped, take the values of the amplitudes with its corresponding time  $\phi_n$ ,  $\phi_{n+1}$ ,  $t_1$ ,  $t_2$ .
- 9. Calculate the period from  $= t_2 t_1$ , and then calculate the damping constant  $\gamma$ .
- 10. Enter the data into the Table 15.1.
- 11. Increase the current by 0.1 A and repeat the steps 7–11.



Fig. 15.4. Setting experiment parameters (left). Start of measurements (right)

<u>Analyse the results</u>. Make <u>conclusions</u> corresponding to the objective of the laboratory work.

### Questions

- 1. What are the simple harmonic oscillations?
- 2. What is amplitude, frequency, cyclic frequency, period of harmonic oscillations?

- 3. Write the equations that describe the harmonic oscillations of the system.
- 4. Definition free, forced and damped rotational oscillations.
- 5. Definition and physical essence of the phenomenon of resonance.
- 6. What is the absolutely solid body?
- 7. What is the difference between the translational and rotational motion of a solid body, give some examples?
- 8. What is the rotation axis of a solid body? What is angular velocity? Which direction has the angular velocity vector?
- 9. How are linear and angular velocities related?
- 10. How many times is the angular velocity of the minute hand of the clock greater than the angular velocity of the clock?
- 11. What is the moment of inertia of the body? Write the equation and units.
- 12. Write the equation of the dynamics of the rotational motion of a solid. Define the quantities in the equation.
## 2. MOLECULAR PHYSICS

### 21. Boyle's Law

<u>Objective</u> Studying Boyle's law. <u>Task</u> Studying the relation between the volumes of an air column as a function of the pressure at constant temperature.

An ideal gas is a gas composed of many randomly moving point particles that are not subject to interparticle interactions. This is an idealized model; it works best at very low pressures (P, the SI unit is pascal (Pa)) and high temperatures, when the gas molecules are far apart and in rapid motion. It is reasonably good at moderate pressures (a few atmospheres) and at temperatures (T, the SI unit is kelvin (K)) well above those at which the gas liquefies:

$$P \cdot V = n \cdot R \cdot T , \qquad (21.1)$$

where V is volume, the SI unit is m<sup>3</sup>, R = 8.31 J/(K·mol) (universal gas constant), and n is quantity of ideal gas in moles. In a given sample of matter n is defined as a ratio  $n = \frac{N}{N_A}$  between the number of molecules/atoms/ions

(N) and the Avogadro constant (N<sub>A</sub>). The value of the Avogadro constant N<sub>A</sub> is defined to be exactly  $6.02214076 \times 10^{23} \text{ mol}^{-1}$ . Also  $= \frac{m}{M}$ , where m is

mass, M is molar mass, the SI unit is kg/mol.

The process in a gas of constant mass which proceeds at constant temperature is termed isothermal. Isothermal processes in gases were studied by the Irish scientist Robert Boyle and the French physicist Edmé Mariotte.

$$P_{1}V_{1} = P_{2}V_{2}$$
(21.2)  
or  
$$\frac{P_{1}}{P_{2}} = \frac{V_{2}}{V_{1}}$$
(21.3)

The equation is the mathematical expression for Boyle's law: for a constant mass of a gas at a constant temperature, its pressure is inversely proportional to its volume. In other words, under the conditions specified, the product of the volume of a gas by its pressure is a constant, that is:

$$PV = const. \tag{21.4}$$

The plot of P vs. V for an isothermal process is a hyperbola and is termed an *isotherm*:



Fig. 21.1. *pV*-Diagram of an ideal gas at constant temperatures  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$ 

<u>Notes</u> 1 Pa= $10^5$  bar, K= $273.15 + ^{\circ}C$ 

## Experimental setup

• A cylindrical tube (height is 20 cm, diameter is 4 cm) containing a certain quantity of air;

- Pressure gauge (-1: 3 bar);
- Neglected volume rubber tube;
- An air tap to reset the apparatus.

## Algorithm of measurements

- 1. Switch off the air tap.
- 2. Generate an under pressure *P* with the piston arm and increase it to maximum value  $\approx$  3 bar.
- 3. Reverse the last step slowly.
- 4. Each time read the height h of the air column, and records it together with P into the Table 21.1. Convert bar to Pa.

*Table 21.1* 

Results of measurements and calculations

<i>h</i> (m)	P (bar)	P (Pa)	$V(\mathrm{m}^3)$
• • •			

- 5. Calculate gas volume:  $= h \cdot \pi \cdot r^2$ , where *r* is the radius of the tube. Record result in the Table 21.1.
- 6. Build a plot P(Pa) vs.  $V(m^3)$ .



Fig. 21.2. The diagrammatic representation of the experimental setup for the investigation of Boyle's law

<u>Analyse the results.</u> Make <u>conclusions</u> corresponding to the objective of the laboratory work.

- 1. What is temperature? How is the Celsius temperature scale built? What is taken as zero on the absolute scale?
- 2. What gas can be considered ideal? Under what conditions are the properties of real gases close to those of an ideal gas?

- 3. What gas laws do you know? What values in them remain constant?
- 4. Depict the isothermal, isobaric, isochoric, and adiabatic processes in the axes *P* vs. *V*, *V* vs. *T*, *P* vs. *T*.
- 5. Write the ideal gas equation.
- 6. What is the mean free path of molecules, and what does it depend on?
- 7. How does the average speed of molecules depend on temperature?
- 8. Kinetic theory of gases.
- 9. Degrees of freedom of a molecule.
- 10. The effect of gas pressure on a wall and the derivation of the formula from the concepts of kinetic theory of gases.

## 22. Pressure-temperature law (Amonton's Law)

#### <u>Objective</u>

Studying the pressure-temperature law.

#### <u>Task</u>

Determination of the dependence of air pressure on temperature at a constant volume.

An ideal gas is a gas composed of many randomly moving point particles that are not subject to interparticle interactions. This is an idealized model; it works best at very low pressures (P, the SI unit is pascal (Pa)) and high temperatures, when the gas molecules are far apart and in rapid motion. It is reasonably good at moderate pressures (a few atmospheres) and at temperatures (T, the SI unit is kelvin (K)) well above those at which the gas liquefies:

$$P \cdot V = n \cdot R \cdot T , \qquad (22.1)$$

where V is volume, the SI unit is m<sup>3</sup>, R = 8.31 J/(K·mol) (universal gas constant), and n is quantity of ideal gas in moles. In a given sample of matter n is defined as a ratio  $n = \frac{N}{N_A}$  between the number of molecules/atoms/ions

(N) and the Avogadro constant (N<sub>A</sub>). The value of the Avogadro constant N<sub>A</sub> is defined to be exactly  $6.02214076 \times 10^{23} \text{ mol}^{-1}$ . Also n =  $m/_M$ , where m is

mass, M is molar mass, the SI unit is kg/mol.

There are three laws of gas: pressure-volume law (Boyle's law), pressure-temperature law (Amonton's law), and volume-temperature law (Gay Lussac's law). The French physicist Guillaume Amonton's built a thermometer based on the fact that the pressure of a gas is directly proportional to its temperature at a constant volume. The relationship between the pressure and the temperature of a gas is therefore known as the pressuretemperature law:

$$P \propto T$$
. (22.2)



Fig. 22.1. Schematic representation of the thermodynamic process

<u>Notes</u> 1 Pa= $10^5$  bar, K= $273.15 + ^{\circ}C$ 

Experimental setup

- Electrical heater;
- Boiler;
- Digital thermometer;
- Pressure gauge.

# Algorithm of measurements

1. Connect the apparatus as shown in Fig. 22.2.



Fig. 22.2. Experimental setup

2. Turn the heater on, after which the gas pressure will begin to increase gradually.

- 3. Record rising in temperature and the corresponding rising in the gas pressure.
- 4. Convert pressure to Pascal and temperature to Kelvin and record it into the Table 22.1.
- 5. Build a plot P(Pa) vs. T(K).

*Table 22.1* 

#### Measurement results

P (psi)	P (Pa)	<i>T</i> (°C)	<i>T</i> (K)
•••			

<u>Analyse the results</u>. Make <u>conclusions</u> corresponding to the objective of the laboratory work.

- 1. What is temperature? How is the Celsius temperature scale built? What is taken as zero on the absolute scale?
- 2. What gas can be considered ideal? Under what conditions are the properties of real gases close to those of an ideal gas?
- 3. What gas laws do you know? What values in them remain constant?
- 4. Depict the isothermal, isobaric, isochoric, and adiabatic processes in the axes P vs. V, V vs. T, P vs. T.
- 5. Write the ideal gas equation.
- 6. What is the mean free path of molecules, and what does it depend on?
- 7. How does the average speed of molecules depend on temperature?
- 8. Kinetic theory of gases.
- 9. Degrees of freedom of a molecule.
- 10. The effect of gas pressure on a wall and the derivation of the formula from the concepts of kinetic theory of gases.

# 23. Volume-temperature law (Gay Lussac's law)

## **Objective**

Studying the volume-temperature law.

#### Tasks

Determination of the temperature dependency of the volume of an air at constant pressure.

Defining the absolute temperature scale by extrapolation towards low temperatures.

An ideal gas is a gas composed of many randomly moving point particles that are not subject to interparticle interactions. This is an idealized model; it works best at very low pressures (P, the SI unit is pascal (Pa)) and high temperatures, when the gas molecules are far apart and in rapid motion. It is reasonably good at moderate pressures (a few atmospheres) and at temperatures (T, the SI unit is kelvin (K)) well above those at which the gas liquefies:

$$P \cdot V = n \cdot R \cdot T , \qquad (23.1)$$

where V is volume, the SI unit is m<sup>3</sup>, R = 8.31 J/(K·mol) (universal gas constant), and n is quantity of ideal gas in moles. In a given sample of matter n is defined as a ratio  $n = \frac{N}{N_A}$  between the number of molecules/atoms/ions

(N) and the Avogadro constant (N<sub>A</sub>). The value of the Avogadro constant N<sub>A</sub> is defined to be exactly  $6.02214076 \times 10^{23} \text{ mol}^{-1}$ . Also  $= \frac{m}{M}$ , where m is

mass, *M* is molar mass, the SI unit is kg/mol.

If one of the quantities P, V or T remains constant, then the other two quantities cannot be varied independently of each other. At a constant pressure P, for example, Gay-Lussac's relationship (volume-temperature law) states:

$$V \propto T$$
. (23.2)

This relationship is confirmed in this experiment by means of a gas thermometer. The gas thermometer consists of a glass capillary open at one end. A certain quantity of air is enclosed by means of a mercury seal. At an outside pressure  $P_0$ , the enclosed air has a volume  $V_0$ . The gas thermometer is placed in a heated bath of a temperature of about  $\vartheta \approx 90^{\circ}$  C which is allowed little by little to cool (Fig. 23.1). The open end of the gas thermometer is subject to the ambient air pressure. Thus, the pressure in the enclosed air column remains constant during the experiment. Its volume is given by:

$$V = \pi \cdot \frac{d^2}{4} \cdot h, \qquad (23.3)$$

where d = 2.7 mm (inside diameter of capillary), and *h* is a height of the gas volume.



Fig. 23.1. Schematic representation of the thermodynamic process

At a constant pressure, the temperature and the volume of an ideal gas are proportional to each other (volume-temperature law).

## Experimental setup

- Gas thermometer;
- Hand vacuum and pressure pump;
- Stand base. V-shaped, 20 cm;
- Stand rod, 47 cm;
- Jaw clamp;
- Hot plate;
- Beaker, 400 ml, hard glass;
- Temperature sensor, NiCrNi;
- Mobile-CASSY.

# Safety notes

! The gas thermometer contains mercury. Handle the gas thermometer with care. Avoid glass breakage.

## Collecting the mercury globules

- Connect the hand vacuum pump to the gas thermometer, and hold the thermometer so that its opening is directed downward (Fig. 23.2).

- Generate maximum underpressure/rarefaction  $\Delta p$  with the hand vacuum pump, and collect the mercury in the bulge (a) so that it forms a drop.

The manometer of the hand vacuum pump displays the underpressure  $\Delta p$  as a negative value.

- If there are mercury globules left, move them into the bulge (a) by slightly tapping the capillary.

A small mercury globule which might have remained at the sealed end of the capillary will not affect the experiment.



Fig. 23.2. Collecting the mercury globules and adjusting the initial gas volume  $V_0$ 

## Adjusting the gas volume $V_0$

- 1. Slowly turn the gas thermometer into its position for use (open end upward) so that the mercury moves to the inlet of the capillary.
- 2. Open the ventilation value (**b**) of the hand vacuum pump carefully and slowly to reduce the underpressure  $\Delta p$  to 0 so that the mercury slides down slowly as one connected seal.

3. Mount the gas thermometer in the stand material and the large test tube like shown in Fig. 23.3, and remove the tubing from the gas thermometer.

If the mercury seal splits due to strong ventilation or vibration, recollect the mercury.

# Measuring the temperature

- Introduce the temperature sensor NiCrNi into the large test tube parallel to the gas thermometer and connect it to the Mobile CASSY.



Fig. 23.3. Schematic representation of the experimental setup. Measuring the temperature with the digital thermometer (**a**) or with Mobile CASSY (**b**), respectively

# Algorithm of measurements

- 1. Heat about 400 ml of water in the beaker to a temperature of about 90°C by means of the hot plate.
- 2. Carefully fill the hot water into the large test tube. Due to the increasing temperature, the initial gas volume will increase.
- 3. Observe the increase of the temperature and wait with reading off the temperature and height until the temperature starts to decrease.
- 4. Measure the temperature  $\vartheta$  (it will be in Celsius and you must convert it to Kelvin) and the height *h* (you must convert it to meter) of the enclosed

gas volume of the gas thermometer while the heat bath (water in the test tube) cools down gradually. Record the results into the Table 23.1.

*Table 23.1* 

## Measurement results

<i>T</i> (°C)	<i>T</i> (K)	<i>h</i> (m)	$V(m^3)$
•••			

5. Build a plot  $V(m^3)$  vs. T(K).

<u>Analyse the results.</u> Make <u>conclusions</u> corresponding to the objective of the laboratory work.

- 1. What is temperature? How is the Celsius temperature scale built? What is taken as zero on the absolute scale?
- 2. What gas can be considered ideal? Under what conditions are the properties of real gases close to those of an ideal gas?
- 3. What gas laws do you know? What values in them remain constant?
- 4. Depict the isothermal, isobaric, isochoric, and adiabatic processes in the axes P vs. V, V vs. T, P vs. T.
- 5. Write the ideal gas equation.
- 6. What is the mean free path of molecules, and what does it depend on?
- 7. How does the average speed of molecules depend on temperature?
- 8. Kinetic theory of gases.
- 9. Degrees of freedom of a molecule.
- 10. The effect of gas pressure on a wall and the derivation of the formula from the concepts of kinetic theory of gases.

#### **3. ELECTRICITY**

#### 31. Resistivity measurement of the wire conductor

*Objective* 

Determination of the resistivity of the material.

Tasks:

Measurement of voltage and current for conductors with different cross-sections and lengths.

In electrical circuits consisting of metal conductors, the voltage drop across the conductor (U) is proportional to the current (I) through it, i.e. Ohm's law:

$$U = R \cdot I, \tag{31.1}$$

where R is an electrical resistance of the conductor.

The resistance R of a conductor of length l and cross-sectional area S can be found as

$$R = \rho \cdot \frac{l}{S}, \qquad (31.2)$$

where  $\rho$  is the specific resistance for the material, it is different from one material to another.



Fig. 31.1. Experimental circuit

In this work, it is necessary to measure the relationship between I and U for conductors made of different materials, with different cross-sections

and lengths. From the obtained U(I) dependences, the resistance is determined for each conductor, the dependence of the resistance on the crosssection and length of the conductor is studied, and the specific resistivity of the conductor material is determined.

The work examines wires made of constantan and brass. Constantan (from Latin *constans* meaning constant, unchanging) is an alloy of copper (Cu) (about 59%), nickel (Ni) (39–41%), and manganese (Mn) (1–2%), which has a weak dependence of electrical resistance on temperature. Brass is a copper-based alloy in which the main additive is zinc (Zn) (up to 50%).



Fig. 31.2. Experimental setup to test Ohm's law

# Experimental setup

- Wire (resistance) set;
- Power supply  $0 \dots 12 V$ ;
- 2 Digital Multimeter;
- Pair of cables 100 cm, red/blue;
- Connecting wire 100 cm black;
- Connecting wire 25 cm black.

# Algorithm of measurements

**Note:** Universal measuring instruments (multimeters) "LDanalog 20" are used as a voltmeter and ammeter. The type of current (alternating current or direct current) and operating mode (current or voltage measurement, scale limit) are selected by turning the switch. Before connecting devices to an electrical circuit, you must select the measurement limit indicated below.

*Task 1*. Measurement of voltage and current for constantan conductors with different cross-sections.

- 1. Connect a voltmeter in parallel to Wire 1 with a diameter (*d*) of 1 mm, then connect a current source and an ammeter to it in series (as shown in Fig. 31.2).
- 2. Turn on the power supply after the engineer or teacher checks it.
- 3. Using the knob on the power source, change the voltage U from 0 to 1.2 V. For each value of U, measure the current I with an ammeter.
- 4. Enter the measurement results into the Table 31.1. Take at least 5 different readings for the ammeter and voltmeter.
- 5. Carry out similar measurements for Wire 2 with d = 0.5 mm and Wire 3 with d = 0.7 mm with the same material (constantan) and the same length l = 1 m. Create a new table similar to Table 31.1 for each wire and enter the experimental results into it.
- 6. On one coordinate system, build a plots U vs. I (voltage (U) in horizontal axis and current (I) in vertical axis) for constantan wires of various diameters and find the slope (slope =  $\frac{\Delta Y}{\Delta X} = \frac{\Delta U}{\Delta I} = R$ ).
- 7. For each wire, determine the resistance *R* from the slope of the corresponding plot and also calculate the cross-section area  $S = \pi d^2/4$ . Enter the obtained values into the Table 31.1.
- 8. Based on the data obtained, build a plot of R vs. 1/S.

*Table 31.1* 

Measurement results

N⁰	$U(\mathbf{V})$	I(A)	$R\left(\Omega ight)$	$S(\mathrm{m}^2)$
1				
2				
•••				

*Task 2*. Measurement of voltage and current for constantan conductors with different lengths.

- 1. Repeat steps 1–4 from *Task 1* for Wire 2 of d = 0.7 mm.
- 2. Create a new table similar to Table 31.1 and enter the experimental results into it.

- 3. Repeat steps 1–4 from *Task 1* for Wire 3 and Wire 4 (with the same d = 0.7 mm and made of the same material) to increase the length of the wire to become  $2 \cdot l (2 \text{ m})$ .
- 4. Create a new table similar to Table 31.1 and enter the experimental results into it.
- 5. On one coordinate system, build a plots U vs. I for constantan wire of length l = 1 m and l = 2 m with a diameter of 0.7 mm. From each plot, determine the resistance R. Enter the obtained values into the table.

*Task 3*. Determination of the resistivity of the materials.

- 1. Repeat steps 1–4 from *Task 1* for Wire 1 (constantan) and Wire 6 (brass).
- 2. Create a new table similar to Table 31.1 for each wire and enter the experimental results into it.
- 3. Build a plot of *U* vs. *I* for constantan and brass wires of the same length (l = 1 m) and diameter (0.5 mm).
- 4. Using the plots, determine *R*.
- 5. Determine  $\rho$  using Eq. (31.2) for brass and constantan.

<u>Analyse the results.</u> Make <u>conclusions</u> corresponding to the objective of the laboratory work.

- 1. What is an electric charge? What is an electric field? Describe their properties.
- 2. What is an electric current? How are the current and current density measured (give the definitions, equations, and units of measurement)? How is the direction of the current defined?
- 3. What are the conditions necessary for the existence of an electric current?
- 4. What materials are conductors? Give some examples. What is the charge carrier in these materials?
- 5. Describe Ohm's law for a circuit without EMF sources (equation, units).
- 6. What is the physical meaning of the conductor resistance? On what factors does resistance depend?
- 7. What is resistivity (also known as specific resistance) of a conductor?
- 8. What devices are used to measure the currents and voltages in a circuit? How should they be connected? Wat is their resistance?
- 9. Derive working equations for finding resistance in methods with accurate measurement of current or voltage.

## 32. Meter bridge

#### <u>Objective</u>

Studying the operational principles underlying a measuring bridge circuit. <u>*Task*</u>

Determination of unknown resistance using the meter bridge.

The Meter bridge is a direct application of the Wheatstone bridge. Both bridges are shown in Figs. 32.1a and 32.1b, respectively.



Fig. 32.1. a) Meter bridge; b) Wheatstone bridge

The meter bridge consists of the unknown resistance R, known resistance Z and a one-meter long steel wire fixed on a wooden table with a ruler representing the two resistances X and Y in the Wheatstone bridge. One of the two terminals of the galvanometer is connected between the R and Z, while the other terminal is connected via a slider to the 1 meter wire, such that it divides that wire into two parts, namely  $l_1$ , and  $l_2$  resembling the two other resistances X, and Y on either sides of the connection point.

At balance condition: (i.e. when the reading of the galvanometer points to zero). The current passing in the two sides of the galvanometer is:

$$I_1 R = I_1 R_{l1} , (32.1)$$

$$I_2 Z = I_2 R_{l2}, (32.2)$$

where  $R_{l1}$  and  $R_{l2}$  are the resistance of the length  $l_1$  and  $l_2$  respectively, which could be determined as follows:

$$R_{l1} = \rho \frac{l_1}{A},$$
(32.3)  

$$R_{l2} = \rho \frac{l_2}{A},$$
(32.4)

where  $\rho$  and *A* are the specific resistance and the cross-sectional area of the wire used. Then, by substituting (32.3), (32.4) into (32.1) and (32.2) and canceling  $I_1$  from both equations, then we have:

$$\frac{R}{Z} = \frac{l_1}{l_2}.$$
 (32.5)

So, the unknown resistance R could be determined by knowing the Z and measuring the  $l_1$  and  $l_2$  values at balance condition.

# Experimental setup

• Meter bridge set of 1 meter-long with sliding contact fixed on a scaled bench;

- Galvanometer with zero in the middle;
- Direct current (DC) power supply;
- Resistance;
- Unknown resistances;
- Connecting leads with banana plugs.

# <u>Algorithm of measurements</u>

- 1. Connect the circuit as shown in Fig. 32.1.
- 2. Connect the resistance *Z* to be  $10 \Omega$ .
- 3. Get the balance condition by moving the slider on the 1 meter wire until there is no deflection in the galvanometer.
- 4. Measure the lengths  $l_1$  and  $l_2$  ( $l_1+l_2 = 1$  m).
- 5. Calculate the unknown resistance R by Eq. (32.5).
- 6. Repeat steps 2-5 by changing the value of the resistance Z.

# Analyse the results.

Make <u>conclusions</u> corresponding to the objective of the laboratory work.

- 1. What is electric current? (equation and definition)
- 2. What types of current are called direct and alternating? Build a plot current vs. time in both cases.
- 3. Which electrical circuit is called a single DC bridge (Wheatstone or meter bridge)? Draw a diagram of the bridge.
- 4. What is called "external forces"? What role do they play? Definition, equation, units of electromotive force (EMF).
- 5. What is a galvanometer, and what is it used for?

# 33. Measuring current and voltage on resistors connected in parallel and in series

### **Objective**

Studying resistors connected in parallel and in series.

### <u>Tasks</u>

Determine the total resistance of resistors connected in parallel.

Determine the total resistance of series-connected resistors.

Kirchhoff's rules are useful techniques to compute the currents and voltages in branched electric circuits:

Junction rule: the algebraic sum of the currents in any junction is zero.

<u>Loop rule</u>: the algebraic sum of the potential differences in any closed loop (including those associated with the electromotive forces and those of the resistive elements) equal zero.

It is necessary to determine "the direction" of the loop. Currents that pass in the same direction, and voltages that cause currents which pass in the same direction, should be considered positive, and in the opposite direction should be considered negative.

We can check the validity of Kirchhoff's rules in electric circuits with resistors connected in parallel and in series.

The voltage U is the same at terminals of every branch of the circuit with parallel connection of the resistors  $R_1$ ,  $R_2$ , ...,  $R_n$ . According to the junction rule, the algebraic sum of the currents  $I_1$ ,  $I_2$ , ...,  $I_n$  equals to the total current through the resistors.

$$I = I_1 + I_2 + \ldots + I_n \,. \tag{33.1}$$

Thus,

$$\frac{U}{R} = \frac{U}{R_1} + \frac{U}{R_2} + \dots + \frac{U}{R_n},$$
 (33.2)

and hence for the total resistance *R*:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}.$$
(33.3)

The current *I* is the same at each point of the electric circuit with series-connected resistors  $R_1, R_2, ..., R_n$ . According to the loop rule, the sum of the potential differences  $U_1, U_2, ..., U_n$  at the terminals of resistors equals to the potential difference of the connected power source.

$$U = U_1 + U_2 + \ldots + U_n \,. \tag{33.4}$$

Thus

$$\cdot R = I \cdot R_1 + I \cdot R_2 + \dots + I \cdot R_n, \qquad (33.5)$$

and the total resistance *R* equals:

Ι

$$R = R_1 + R_2 + \dots + R_n \,. \tag{33.6}$$

Experimental setup

- Plug-in board DIN A4;
- Resistor 220  $\Omega$ , 2 W;
- Resistor 330  $\Omega$ , 2 W;
- Resistor 470 Ω, 1.4 W;
- Resistor 1 k $\Omega$ , 2 W;
- Resistor 5.6 k $\Omega$ , 2 W;
- Resistor 10 k $\Omega$ , 0.5 W;
- Resistor 100 k $\Omega$ , 0.5 W;
- Set 10 bridging plugs;
- direct current (DC) power supply,  $0 \pm 15$  V;
- Multimeter LDanalog 20 (2 pts.);
- Pairs of cables 50 cm, red/blue (3 pts.).

# Algorithm of measurements

Multimeters LDanalog 20 should be used as a voltmeter and ammeter in the experiment. Before connecting devices to the circuit, select the measurement limits as specified below. The operating mode (current or voltage measurement, ranges, alternating current (AC) or direct current (DC) type) can be set by turning the switch.

Task 1. Determine the total resistance of series-connected resistors

- 1. Assemble the electric circuit as shown in Fig. 33.1. Please note that the wires to the power source must be connected to the «+» and «0» sockets on the front panel (do not use «-» socket). The teacher/engineer should choose the resistors to measure. Since only two multimeters are used, the voltage drop (task 1) or current (task 2) measuring across the resistors is performed by the same device connected to the appropriate nodes of the circuit.
- 2. Set the output voltage U = 10 V using the appropriate button on the power source. You are authorized to turn on the power source only after the electric circuit has been checked by a teacher/engineer. Turn on the DC power source.

3. Measure and record down the total current  $I = I_{ex}$  and potential difference U. Determine the experimental value of the circuit resistance  $R_{ex} = \frac{V_{total}}{I_{total}}$ .



Fig. 33.1. Setup for series connection resistors



Fig. 33.2. Setup for parallel connection resistors

4. Calculate the theoretical value of the resistance R, from Eq. (33.6).

*Table 33.1* 

# Measurement results for series connection

N⁰	$R_1(\Omega)$	$R_2(\Omega)$	$R_3(\Omega)$	$R_{\rm total}(\Omega)$	$U_{\text{total}}(V)$	$I_{\text{total}}(A)$
•••						

- 5. Start to change the voltage from the power supply. Measure and record down potential difference and current at least 5 times.
- 6. Enter your data into Table 33.1.

Task 2. Determine the total resistance of resistors connected in parallel

- 1. Assemble the electric circuit as shown in Fig. 33.2.
- 2. Calculate the theoretical value of the resistance R from Eq. (33.3).
- 3. Start to change the voltage from the power supply. Measure and record down potential difference and current at least 5 times.
- 4. Enter your data into Table 33.2.

*Table 33.2* 

Measurement results for parallel connection

N⁰	$R_1(\Omega)$	$R_2(\Omega)$	$R_3(\Omega)$	$R_{\rm total}(\Omega)$	$U_{\text{total}}(V)$	$I_{\text{total}}(A)$

- 5. Take only one resistor from those used by choice. Start to change the voltage from the power supply. Measure and record down potential difference and current at least 5 times.
- 6. Enter your data into Table 33.3.
- 7. On one coordinate system, build a plots U vs. I (voltage (U) in horizontal axis and current (I) in vertical axis) for all three tables.
- 8. Find the slope for each curve.

*Table 33.3* 

Measurement results for one resistor

N⁰	$N_{2}$ $R(\Omega)$		I(A)
•••			

9. Calculate the error:

$$Error = \frac{R_{theoretical} - R_{experimental}}{R_{theoretical}} * 100 = \_\%$$

<u>Analyse the results.</u> Make <u>conclusions</u> corresponding to the objective of the laboratory work.

- 1. What is an electric charge? What is current and current density? How is the direction of current defined?
- 2. Conditions necessary for the existence of a current.
- 3. Why cannot the electrostatic field of charged particles maintain constant current in a circuit?
- 4. What is called "external forces"? What role do they play? Definition, equation, units of electromotive force (EMF).
- 5. Ohm's law for a circuit with a power source(s). How is the sign of the EMF defined in this equation?
- 6. Resistors connected in parallel. Currents, voltages, resistance.
- 7. Resistors connected in series. Currents, voltages, resistance.
- 8. Kirchhoff's rules. What is the physical background of the junction rule and the loop rule?
- 9. Calculate the parameters of a circuit given by the teacher using the Kirchhoff's rules.

# 34. Charging and discharging of a capacitor

## **Objective**

Studying charging and discharging of a capacitor.

Tasks

Investigating the behavior of the voltage at a capacitor when a direct current (DC) voltage is switched on and off.

Determining time constant  $\tau$ .

In a DC circuit, a capacitor represents an infinite resistance. Current passes through during the closing and opening of the circuit. When the circuit is closed, the capacitor to be charged until the applied voltage is reached. Correspondingly, the capacitor is discharged via a resistor when the circuit is opened. The behavior of the voltage at the capacitor can be described by means of an exponential function.



Fig. 34.1. Time dependence of voltage and current during charging (left) and discharging (right) of a capacitor

The process of capacitor discharge can be described by the equation:

$$U(t) = U_0 \cdot exp\left(-\frac{t}{\tau}\right), \qquad (34.1)$$

with  $\tau$  is the time constant or decay time  $\tau \equiv R \cdot C$ . It is the time after which the voltage has dropped to the value  $U_0/e$ ,  $e \approx 2.71828$  is the base of the natural logarithm and exponential function (Euler's number).

Dependence of electric potential drop at the terminals of the capacitor on the charging time is:

$$U(t) = U_0 \cdot \left(1 - exp\left(-\frac{t}{\tau}\right)\right). \tag{34.2}$$



Fig. 34.2. Experimental circuit

# Experimental setup

- Plug-in board;
- Resistor 1 M $\Omega$ ;
- Capacitors 470 µF;
- Stopwatch;
- Pair of cables, 100 cm, blue and red;
- Pair of cables, 50 cm, blue and red.

# Algorithm of measurements

*Task 1*. Investigating the charging processes of a capacitor

- 1. Connect the power supply with the capacitor and resistant in series and the capacitor in parallel with the voltmeter (Fig. 34.2).
- 2. Turn on the power supply (5 V) and at the same time start the stopwatch. Record the voltage on the voltmeter every 10 seconds, take 10 measurements of time and voltage until the readings are equal to the power supply readings.
- 3. Enter the measurement results into the Table 34.1.

*Table 34.1* 

Measurement results for charging the capacitor

<i>t</i> (s)	<i>U</i> (V)
10	
20	
••••	

Task 2. Investigating the discharging processes of a capacitor

- 1. Reset the stopwatch.
- 2. Turn off the power supply and at the same time start the stopwatch. Record the voltage on the voltmeter every 10 seconds, take 10 measurements.
- 3. Enter the measurement results into the Table 34.2.

*Table 34.2* 

Measurement results for discharging the capacitor

<i>t</i> (s)	<i>U</i> (V)	$\ln(\frac{U}{U_0})$
10		
20		
••••		

- 4. Plot graphs of voltage U(V) vs. t(s).
- 5. Plot the dependence of  $\ln(\frac{U}{U_0})$  vs. t(s) and find slope  $slope = \frac{\Delta y}{\Delta x}$ .
- 6. From Eq. 34.1 derive the relationship between the voltage on the power supply and the voltage on the voltmeter:

$$\frac{U(t)}{U_0} = \exp\left(-\frac{t}{\tau}\right). \tag{34.3}$$

Take natural logarithm on both sides:

$$\ln\left(\frac{U(t)}{U_0}\right) = -\frac{t}{\tau}.$$
(34.4)

Thus,

$$slope = \frac{\Delta y}{\Delta x} = \frac{\Delta \ln\left(\frac{U(t)}{U_0}\right)}{\Delta t} = -\frac{1}{RC} = -\frac{1}{\tau}.$$
(34.5)

7. Calculate time constant  $(\tau)$  theoretically and enter results into the Table 34.3.

*Table 34.3* 

Calculation of the theoretical time constant

$R\left(\Omega ight)$	$C(\mathbf{F})$	$\tau$ (s)

8. Calculate error:

$$Error = \frac{\tau_{theoreatical} - \tau_{experimental}}{\tau_{experimental}} * 100\% = \_\%$$

<u>Analyse the results.</u> Make <u>conclusions</u> corresponding to the objective of the laboratory work.

- 1. What is electric current? What current is called direct?
- 2. Definition of electrical capacitance. In which units is it measured in the International System of Units (SI)?
- 3. Formula for determining the capacitance of a flat capacitor.
- 4. What is the capacitance of a system of capacitors when they are connected in series and in parallel?
- 5. What is called the time constant of a circuit, what is its dimension? How does the current change over time when charging and discharging a capacitor?
- 6. How do the voltages on the capacitor and the resistance of the circuit change when charging and discharging the capacitor?
- 7. Draw graphs of voltage dependence during the process of charging and discharging of a capacitor.

# 35. Circuits with coils and ohmic resistors

# <u>Objective</u>

Studying the circuits with coils and ohmic resistors.

Tasks

Determination of the total impedance and the phase shift in a series connection of a coil and a resistor.

Determination of the total impedance and the phase shift in a parallel connection of a coil and a resistor.

If an alternating voltage

 $U = U_0 \cdot \cos(\omega \cdot t)$ , with  $\omega = 2\pi \cdot f$ , (35.1)

is applied to a coil with the inductance L, the current passing through the coil is

$$I = \frac{U_0}{\omega \cdot L} \cdot \cos\left(\omega \cdot t - \frac{\pi}{2}\right). \tag{35.2}$$

Therefore, an inductive reactance:

$$X_L = \omega \cdot L \tag{35.3}$$

is assigned to the coil, and the voltage is said to be phase shifted with respect to the current by  $90^{\circ}$  (Fig. 35.1). The phase shift is often represented in a vector diagram.



Fig. 35.1. Alternating current circuit with a coil (circuit diagram, vector diagram and U(t), I(t) diagram)

Series connection

If the coil in connected in series with an ohmic resistor, the same current passes through both components. This current can be written in the form  $I = I_0 \cdot \cos(\omega \cdot t - \varphi_S),$  (35.4) where  $\varphi_S$  is unknown for the time being. Correspondingly, the voltage drop is

$$U_R = R \cdot I_0 \cdot \cos(\omega \cdot t - \varphi_s), \qquad (35.5)$$

at the resistor and

$$U_L = X_L \cdot I_0 \cdot \cos\left(\omega \cdot t - \varphi_s + \frac{\pi}{2}\right), \qquad (35.6)$$

at the coil. The sum of these two voltages is

$$U_S = \sqrt{R^2 + X_L^2} \cdot I_0 \cdot \cos(\omega \cdot t), \qquad (35.7)$$

if  $\varphi_S$  fulfills the condition

$$\tan \varphi_S = \frac{X_L}{R}, \qquad (35.8)$$

 $U_{\rm S}$  is equal to the voltage U applied, and therefore

$$U_0 = \sqrt{R^2 + X_L^2} \cdot I_0 , \qquad (35.9)$$

i.e. the series connection of an ohmic resistor and a coil can be assigned the impedance

$$Z_S = \sqrt{R^2 + X_L^2}.$$
 (35.10)

In this arrangement, the voltage is phase-shifted with respect to the current by the angle  $\varphi_S$  (Fig. 35.2).



Fig. 35.2. Alternating current circuit with a coil and an ohmic resistor in series connection (circuit diagram, vector diagram and U(t), I(t) diagram)

## Parallel connection

If the coil is connected in parallel to the ohmic resistor, the same voltage is applied to both of them. The voltage has, for example, the shape given in Eq. (35.1). The current passing through the ohmic resistor is

$$I_R = \frac{U_0}{R} \cdot \cos(\omega \cdot t), \qquad (35.11)$$

whereas the current passing through the capacitor is

$$I_L = \frac{U_0}{X_L} \cdot \cos\left(\omega \cdot t - \frac{\pi}{2}\right). \tag{35.12}$$

The sum of the two currents is

$$I_{P} = \sqrt{\frac{1}{R^{2}} + \frac{1}{X_{L}^{2}}} \cdot U_{0} \cdot \cos(\omega \cdot t - \varphi_{P}), \quad (35.13)$$

with  $tan\varphi_P = \frac{R}{X_L}$ .

It corresponds to the total current drawn from the voltage source. Hence, the parallel connection of an ohmic resistor and a coil can be assigned an impedance  $Z_P$ , for which the relation holds. In this arrangement, the voltage is phase-shifted with respect to the current by the angle  $\varphi_S$  (Fig. 35.3).

$$\frac{1}{Z_P} = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}.$$
 (35.14)



Fig. 35.3. Alternating current circuit with a coil and an ohmic resistor in series connection (circuit diagram, vector diagram and U(t), I(t) diagram)

In the experiment, the current I(t) and the voltage U(t) are measured as time-dependent quantities in an alternating current (AC) circuit by means of a two-channel oscilloscope. A function generator is used as a voltage source with variable amplitude  $U_0$  and variable frequency f. From the measured quantities, the magnitude of the total impedance Z and the phase shift  $\varphi$  between the voltage and the current are determined.

Experimental setup

- Plug-in board A4;
- Resistor 1  $\Omega$ , 2 W;
- Resistor 100 Ω, 2 W;
- Capacitor 0.1 µF, 100 V;
- Capacitor 1 µF, 100 V;

- Capacitor 10 µF, 100 V;
- Function generator S 12;
- Two-channel oscilloscope 303;
- Screened cables BNC/4 mm;
- Connecting leads.

# Assembling the setup

The experimental setup is illustrated in Fig. 35.4.



Fig. 35.4. Experimental setup for determining the impedance in circuits with coils and ohmic resistors in series connection

- 1. Connect the function generator as an alternating current (AC) voltage source, and select the shape  $\sim$
- 2. Connect the channel I of the oscilloscope to the output of the function generator, and feed the voltage drop at the measuring resistor 1  $\Omega$  into the channel II.
- 3. Press the DUAL pushbutton at the oscilloscope and select AC for the coupling and the trigger.



Fig. 35.5. Experimental setup for determining the impedance in circuits with coils and ohmic resistors in parallel connection

# Algorithm of measurements

- 1. Connect the coil with 1000 turns as an inductance in series with the 100  $\Omega$  resistor.
- 2. Switch the function generator on by plugging in the plug-in power supply, and adjust a frequency of 10000 Hz (T = 0.1 ms).
- 3. Select an appropriate time-base sweep at the oscilloscope.
- 4. Adjust an output signal of 5 V.
- 5. Read the amplitude  $U_{\rm m}$  of the signal in the channel II of the oscilloscope and enter it into the Table 35.1 as current  $I_0 = \frac{U_m}{10}$ .
- 6. Read the interval of time  $(\Delta t)$  between the zero passages of the two signals.
- 7. Replace the coil by the coil with 500 turns, and repeat the measurement.
- 8. One after another, connect the two coils in parallel with the 100  $\Omega$  resistor (Fig. 35.5), and repeat the measurement.
- 9. Adjust other frequencies according to the Table 35.1 and repeat the measurements.

*Table 35.1* 

			Series co	Series connection		onnection
$f(\mathrm{Hz})$	<i>T</i> (s)	Ν	$I_0(\mathbf{A})$	$\Delta t$ (s)	$I_0(\mathbf{A})$	$\Delta t$ (s)
10000	0.0001	1000				
10000	0.0001	500				
5000	0.0002	1000				
5000	0.0002	500				
2000	0.0005	1000				
2000	0.0005	500				
1000	0.001	1000				
1000	0.001	500				
500	0.002	1000				
500	0.002	500				
200	0.005	1000				
200 0.003	0.005	500				
100	0.01	1000				
50	0.02	1000				

# Results of measurements

## Analysis of results

1. Calculate the phase shift values for all measured values of  $\Delta t$ :

$$\varphi = 360 \cdot \frac{\Delta t}{T}.$$

2. Calculate for all measured values the current amplitude/inductive reactance value  $X_L = 2\pi \cdot f \cdot L$  (coil inductance L = 4.25 mH at N = 500turns and L = 17 mH at N = 1000) and circuit impedance:

$$Z = \frac{U_0}{I_0}.$$

- 3. Enter the calculation results into the Table 35.2.
- 4. Draw conclusion.

# *Table 35.2*

		Series co	nnection	Parallel co	onnection
f(Hz)	N	$Z\left( \Omega ight)$	$\varphi$	$Z(\Omega)$	$\varphi$
10000	1000				
10000	500				
5000	1000				
5000	500				
2000	1000				
2000	500				
1000	1000				
1000	500				
500	1000				
500	500				
200	1000				
200	500				
100	1000				
50	1000				

Values of Z and  $\phi$  calculated from the data of the Table 35.1

- 1. Build a plots Z vs. X for connecting an inductor and a resistor in series and in parallel. Apply logarithmic scales for Z and X.
- 2. Build a plots f vs. X, for series and parallel connection of an inductor and a resistor. Apply a logarithmic scale for X and a linear scale for f.
- 3. Explain your results.
- 4. Draw <u>conclusion</u>.

- 1. What is alternating current? Build plots *I* vs. *t* and *U* vs. *t*. Write down a law of change in voltage and current.
- 2. How are voltage and current related in the resistor (for an alternating current circuit)?
- 3. What is a capacitor? What determines its capacity (equation, unit)? Why can't direct current pass through a capacitor?
- 4. What is an inductor? What determines its inductance (unit)? Why does a self-induction EMF appear in it when the current in the coil changes?

- 5. Which electric circuit is called an *L*-*R*-*C* series circuit? Where is the energy of the electromagnetic field stored in the *L*-*R*-*C* series circuit?
- 6. Explain why harmonic undamped oscillations of charge and current occur in the L-C series circuit. According to which law do the charge of the capacitor and the current in the inductor change with time?
- 7. How does the oscillation period in an *L*-*R*-*C* series circuit depend on the magnitude of the electric capacitance and inductance of the elements?
- 8. Describe the phenomenon of resonance in an *L*-*R*-*C* series circuit. How is the phenomenon of resonance used in radio engineering? Draw a resonance curve for two different resistance values.

# 36. Circuits with capacitors and ohmic resistors

## **Objective**

Studying the circuits with capacitors and ohmic resistors.

Tasks

Determination of the total impedance and phase shift when connecting a capacitor and resistor in series.

Determination of the total impedance and phase shift when connecting a capacitor and resistor in parallel.

If an alternating voltage

 $U = U_0 \cdot \cos(\omega \cdot t)$  with  $\omega = 2\pi \cdot f$ , (36.1)

is applied to a capacitor with the capacitance C, the current passing through the capacitor is

$$I = U_0 \cdot \omega \cdot C \cdot \cos\left(\omega \cdot t + \frac{\pi}{2}\right). \tag{36.2}$$

Therefore, a capacitive reactance

$$X_C = \frac{1}{\omega \cdot C},\tag{36.3}$$

is assigned to the capacitor, and the current is said to be phase-shifted with respect to the voltage by  $90^{\circ}$  (Fig. 36.1). The phase shift is often represented in a vector diagram.



Fig. 36.1. Alternating current circuit with a capacitor (circuit diagram, vector diagram and U(t), I(t) diagram)

## Series connection

If the capacitor in connected in series with an ohmic resistor, the same current passes through both components. This current can be written in the form
$$I = I_0 \cdot \cos(\omega \cdot t + \varphi_S), \qquad (36.4)$$

where  $\phi_S$  is unknown for the time being. Correspondingly, the voltage drop is

$$U_R = R \cdot I_0 \cdot \cos(\omega \cdot t + \varphi_S), \qquad (36.5)$$

at the resistor and

$$U_C = X_C \cdot I_0 \cdot \cos\left(\omega \cdot t + \varphi_S - \frac{\pi}{2}\right), \tag{36.6}$$

at the capacitor. The sum of these two voltages is

$$U_{S} = \sqrt{R^{2} + X_{C}^{2}} \cdot I_{0} \cdot \cos(\omega \cdot t), \qquad (36.7)$$

if  $\varphi_S$  fulfills the condition

$$\tan \varphi_{\rm S} = \frac{X_C}{R},\tag{36.8}$$

 $U_S$  is equal to the voltage U applied, and therefore

$$U_0 = \sqrt{R^2 + X_c^2} \cdot I_0 , \qquad (36.9)$$

i.e. the series connection of an ohmic resistor and a capacitor can be assigned the impedance

$$Z_S = \sqrt{R^2 + X_C^2}.$$
 (36.10)

In this arrangement, the current is phase-shifted with respect to the voltage by the angle  $\varphi_S$  (Fig. 36.2).



Fig. 36.2. Alternating current circuit with a capacitor and an ohmic resistor in series connection (circuit diagram, vector diagram and U(t), I(t) diagram)

#### Parallel connection

If the capacitor is connected in parallel to the ohmic resistor, the same voltage is applied to both of them. The voltage has, for example, the shape given in Eq. (36.1). The current passing through the ohmic resistor is

$$I_R = \frac{U_0}{R} \cdot \cos(\omega \cdot t), \qquad (36.11)$$

whereas the current passing through the capacitor is

$$I_C = \frac{U_0}{X_C} \cdot \cos(\omega \cdot t + \frac{\pi}{2}) . \qquad (36.12)$$

The sum of the two currents is

$$I_P = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}} \cdot U_0 \cdot \cos(\omega \cdot t + \varphi_P), \qquad (36.13)$$
$$= \frac{R}{2}$$

with  $tan \varphi_P = \frac{R}{X_C}$ .

It corresponds to the total current drawn from the voltage source. Hence, the parallel connection of an ohmic resistor and a capacitor can be assigned an impedance  $Z_P$ , for which the relation

$$\frac{1}{Z_P} = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}}$$
(36.14)

holds. In this arrangement, the current is phase-shifted by the angle  $\varphi_P$  with respect to the voltage (Fig. 36.3).



Fig. 36.3. Alternating current circuit with a capacitor and an ohmic resistor in parallel connection (circuit diagram, vector diagram and U(t), I(t) diagram)

In the experiment, the current I(t) and the voltage U(t) are measured as time-dependent quantities in an alternating current (AC) circuit by means of a two-channel oscilloscope. A function generator is used as a voltage source with variable amplitude  $U_0$  and variable frequency f. From the measured quantities, the magnitude of the total impedance Z and the phase shift  $\varphi$  between the current and the voltage are determined.

### Experimental setup

- Plug-in board A4;
- Resistor 1  $\Omega$ , 2 W;
- Resistor 100 Ω, 2 W;
- Capacitor 0.1 µf, 100 V;

- Capacitor 1 µf, 100 V;
- Capacitor 10 µf, 100 V;
- Function generator;
- Two-channel oscilloscope;
- Screened cables BNC/4 mm;
- Connecting leads.

# Assembling the setup

The experimental setup is illustrated in Fig. 36.4.



Fig. 36.4. Experimental setup for determining the impedance in circuits with capacitors and ohmic resistors in series connection

- 1. Connect the function generator as an AC voltage source, and select the shape  $\sim$ .
- 2. Connect the channel I of the oscilloscope to the output of the function generator, and feed the voltage drop at the measuring resistor into the channel II.
- 3. Press the DUAL pushbutton at the oscilloscope and select AC for the coupling and the trigger.

### Algorithm of measurements

- 1. Connect the 10  $\mu$ F capacitor as a capacitance in series with the 100  $\Omega$  resistor (Fig. 36.4).
- 2. Switch the function generator on by plugging in the plug-in power supply, and adjust to a frequency of 2000 Hz (T = 0.5 ms).
- 3. Select an appropriate time-base sweep at the oscilloscope.
- 4. Adjust an output signal of 5 V.
- 5. Read the amplitude  $U_m$  of the signal in channel II of the oscilloscope, and enter it into the Table 36.1 as current  $I_0 = \frac{U_m}{10}$ .
- 6. Read the time interval ( $\Delta t$ ) between the zero passages of the two signals.



Fig. 36.5. Experimental setup for determining the impedance in circuits with capacitors and ohmic resistors in parallel connection

- 7. Connect the 10  $\mu$ F capacitor in parallel to the 100  $\Omega$  resistor (Fig. 36.5).
- 8. Repeat the measurement.
- 9. Repeat the measurements with the 1  $\mu$ F capacitor and with the 0.1  $\mu$ F capacitor.
- 10. Adjust other frequencies according to Table 36.1, and repeat the measurements.

*Table 36.1* 

			Series connection		Parallel c	onnection
$f(\mathrm{Hz})$	<i>T</i> (s)	<i>C</i> (µF)	$I_0(\mathbf{A})$	$\Delta t$ (s)	$I_0(\mathbf{A})$	$\Delta t$ (s)
		10				
2000	0.0005	1				
		0.1				
		10				
1000	0.001	1				
		0.1				
500	0.002	10				
500	0.002	1				
200	0.005	10				
200	200 0.005	1				
100	0.01	10				
100	0.01	1				
50	0.02	10				

# Results of measurements

# Analysis of results

1. Calculate the phase shift ( $\varphi$ ) values for all measured values of  $\Delta t$ :

$$\varphi = 360 \cdot \frac{\Delta t}{T}.$$

2. Calculate for all measured values of current amplitude  $I_0$  the values of capacitance resistance

$$X_C = \frac{1}{2\pi f \cdot C} \, .$$

3. and the total impedance of the circuit

$$z=\frac{U_0}{I_0}.$$

4. Enter the calculation results into the Table 36.2.

*Table 36.2* 

		Series connection		Parallel connection		
f(Hz)	C (µF)	$X_{C}(\Omega)$	$Z(\Omega)$	φ	$Z(\Omega)$	φ
	10					
2000	1					
	0.1					
	10					
1000	1					
	0.1					
500	10					
500	1					
200	10					
200	1					
100	10					
	1					
50	10					

# Values of Z and $\varphi$ calculated from data of the Table 36.1

5. Build a plots Z vs.  $X_C$  for connecting a capacitor and a resistor in series and in parallel.

- 6. Apply logarithmic scales for Z and  $X_C$ .
- 7. Build a plots  $\varphi$  vs.  $X_C$  for a series and parallel connection of a capacitor and a resistor. Apply a logarithmic scale for  $X_C$  and a linear one for  $\varphi$ .
- 8. Explain your results. Draw <u>conclusion</u>.

# Questions

- 1. What is alternating current? Build plots *I* vs. *t* and *U* vs. *t*. Write down a law of change in voltage and current.
- 2. How are voltage and current related in the resistor (for an alternating current circuit)?
- 3. What is a capacitor? What determines its capacity (equation, unit)? Why can't direct current pass through a capacitor?
- 4. What is an inductor? What determines its inductance (unit)? Why does a self-induction EMF appear in it when the current in the coil changes?

- 5. Which electric circuit is called an *L*-*R*-*C* series circuit? Where is the energy of the electromagnetic field stored in the *L*-*R*-*C* series circuit?
- 6. Explain why harmonic undamped oscillations of charge and current occur in the L-C series circuit. According to which law do the charge of the capacitor and the current in the inductor change with time?
- 7. How does the oscillation period in an L-R-C series circuit depend on the magnitude of the electric capacitance and inductance of the elements?
- 8. Describe the phenomenon of resonance in an *L-R-C* series circuit. How is the phenomenon of resonance used in radio engineering? Draw a resonance curve for two different resistance values.

### 37. Circuits with capacitors and coils

#### **Objective**

Studying the circuits with capacitors and coils.

#### Tasks

Determination of the resonance frequency with circuits of capacitors and coils in series and parallel.

Determination of the alternating current resistance depending on the frequency with circuits of capacitors and coils in series and parallel. Observe the phase shift.

In order to examine the alternating current (AC) resistance, a circuit is constructed with a coil (inductance L), a capacitor (capacitance C), and a resistor R. This type of circuit is also known as an R-C-L resonant circuit. If a charged capacitor is discharged within it via a coil, then the voltage at the capacitor does not drop to zero exponentially, but instead oscillates. The energy is thereby transferred between electrical and magnetic fields. The resonant frequency:

$$f_R = \frac{1}{2\pi\sqrt{LC'}},\tag{37.1}$$

with L is an inductance, C is a capacitance.

In the test, the resonant circuit is tested with the capacitance and inductance in series and in parallel. For this purpose, the voltage drop is measured across a resistor with an oscilloscope, and thereby the current strength is dependent on the frequency applied.

The following applies to the AC resistance of the LC part when wired in series:

$$Z_S = \left| 2\pi f L - \frac{1}{2\pi f C} \right|,\tag{37.2}$$

and in parallel:

$$\frac{1}{Z_P} = \left| \frac{1}{2\pi fL} - 2\pi fC \right|.$$
(37.3)

By applying the resonant frequency  $f_R$ , the right side of the equation becomes zero, i.e., the resistance disappears in the series circuit and becomes infinite in the parallel circuit.

The series circuit is known as a filter circuit or a band-pass filter. Only a certain frequency band of the input signal is able to pass the filter. Frequencies outside the resonant frequency are attenuated accordingly.

The parallel circuit is used as a trap circuit or band-stop. Resonant frequencies are blocked. All other frequencies are able to pass the filter.

For the phase shift  $\phi$  with RCL series connection (Fig. 37.1), the following applies:

$$\tan \varphi = \frac{2\pi fL - \frac{1}{2\pi fC}}{R}.$$
(37.4)

In resonant frequency, the phase shift is  $0^{\circ}$ . With very low frequencies, it runs to  $-90^{\circ}$ , and with very high frequencies, it runs to  $+90^{\circ}$ .

With a parallel connection, the following applies for the phase shift ( $\phi$ ) (Fig. 37.2):

$$\tan \varphi = \frac{1}{R(\frac{1}{2\pi fL} - 2\pi fC)}.$$
 (37.5)

With very low frequencies and with very high frequencies, the phase shift tends to 0°. When approaching the resonant frequency, it tends to  $+90^{\circ}$  if the frequency is slightly less than the resonant value, or  $-90^{\circ}$  if the frequency is slightly higher than the resonant value. When the frequency passes through the resonant value, the phase abruptly changes by 180 degrees (from -90 to +90).

### <u>Experimental setup</u>

- Plug-in board;
- Resistor 10  $\Omega$ ;
- Resistor 100  $\Omega$ ;
- Capacitor 1  $\mu$ F;
- Capacitor 4.7  $\mu$ F;
- Coil, 500 windings;
- Coil, 1000 windings;
- Function generator;
- Dual channel oscilloscope 303;
- 2 measuring cable bnc/4 mm;
- Pair of cables, red and blue, 100 cm.

Task 1. Determining the resonant frequency

## Series connection

Assemble the electrical circuit according to Fig. 37.1. Use LC combination per Table 37.1 and a resistor  $R = 100 \Omega$ .

Attention! It is important to use only the specified resistor because resonance in the coil and capacitor can produce significantly higher voltages than the input voltage. This is possible when connecting a capacitor and inductor in series.

- 1. First set the frequency generator to 100 Hz (Sinus) and set a peak voltage of  $U_E = 6$  V with the aid of the oscilloscope.
- 2. During the experiment, the timebase of the oscilloscope must be aligned with the respective frequency setting.
- 3. Increase the frequency until  $U_R$  is maximum (i.e. Z minimum).
- 4. Enter the frequency into the Table 37.1.

*Table 37.1* 

D		C	•	•	•	, •
к	econant	treatien	CIES	111	ceriec	connection
T	Coonant	nequen	CIUS	111	301103	connection
		1				

	f (Hz)				
Inductor, L	$C = 1 \ \mu F$	$C = 4.7 \ \mu F$			
500 windings					
1000 windings					

5. Measure using further LC combinations in accordance with Table 37.1.

# Parallel connection

- 1. Test apparatus per Fig. 37.2. LC combination per Table 37.2. Resistor  $R = 100 \Omega$ .
- 2. Increase the frequency until  $U_R$  is minimum (i.e. Z maximum).
- 3. Enter the frequency into the Table 37.2.
- 4. Measure using further LC combinations in accordance with Table 37.2.

## *Table 37.2*

	C	•	•	1	1 1	, •
Reconant	treatienc	10C	1n	naral	Iel	connection
Resonant	nequence		111	parar	IUI	connection
	L			T		

	$f(\mathrm{Hz})$				
Inductor, L	$C = 1 \ \mu F$	$C = 4.7 \ \mu F$			
500 windings					
1000 windings					



Fig. 37.1. Experimental setup for determining resonant frequency in series connection



Fig. 37.2. Experimental setup for determining resonant frequency in parallel connection

*Task 2*. Determining the AC resistance Series connection

Series connection

- 1. First rearrange the test apparatus into series connection with the coil 1000 windings, the capacitor 1  $\mu$ F and the resistor 100  $\Omega$ .
- 2. Increase the frequency from 50 Hz to 20 000 Hz in stages. Enter frequencies f and peak voltages  $U_R$  into the Table 37.3.
- 3. Repeat the measurement with the resistor 10  $\Omega$ .

*Table 37.3* 

### Series connection

$f(\mathrm{Hz})$	U <sub>R</sub>	(V)
	$R = 100 \ \Omega$	$R = 10 \ \Omega$
50		
200		
500		
1000		
1220		
1500		
2000		
5000		
10000		
20000		

### Parallel connection

Repeat the test with the circuit in parallel connection and enter the measured values into the Table 37.4.

*Table 37.4* 

### Parallel connection

$f(\mathbf{U}_{\mathbf{Z}})$	$U_R$ (V)					
<i>J</i> (ΠΖ)	$R = 100 \ \Omega$	$R = 10 \ \Omega$				
50						
200						
500						
1000						
1220						
1500						
2000						
5000						
10000						
20000						

Task 3. Observing the phase relation

Series connection

- 1. First rearrange the test apparatus into series connection with the coil 1000 windings, the capacitor 1  $\mu$ F and the resistor 100  $\Omega$ .
- 2. Set the resonant frequency and observe the phase.
- 3. Increase the frequency slightly and then decrease once more. When doing so, observe the phase relation.
- 4. Set a low frequency ( $\approx 100 \text{ Hz}$ ), followed by a high frequency, and observe the phase relation. If necessary, increase the amplification of the output signal.

Parallel connection

Repeat the test with parallel connection.

# Analysis of results

The equation for the calculation of the inductance of the coil is

$$L = \mu_0 N^2 \frac{A}{l}$$

with  $\mu_0 = 1.26 \cdot 10^{-6} \frac{\text{H}}{\text{m}}$ ; *N* is a number of windings and  $\frac{A}{l} \approx 1.4 \cdot 10^{-3}$  m with the coil formers used.

This results in:  $L_{500Wdg} \approx 4.4 \text{ mH}$ ;  $L_{1000Wdg} \approx 17.6 \text{ mH}$ .

# Calculation of resonant frequency

Calculate the theoretical values of the resonant frequency according to Eq. (37.1) for the values of inductance and capacitance used in the first task and compare with the experimentally measured ones.

# AC resistance determination

Calculate the AC resistance of the series and parallel circuit (Table 37.4) using the measured voltage values and the theoretical values according to Eqs. (37.2) and (37.3) and fill in the Table 37.4.

	Series connection			Parallel connection			
	$R = 100 \ \Omega$	<i>R</i> = 10	Theore-	$R = 100 \ \Omega$	$R = 10 \Omega$	Theore-	
		Ω	tical			tical	
			value			value	
$f(\mathrm{Hz})$	$Z = \frac{(U_E - U_E)}{U_R}$	$\frac{R^{)\cdot R}}{\Omega}(\Omega)$	$Z_S$	$Z = \frac{(U_E - U_E)}{U_E}$	$\frac{U_R)\cdot R}{R}(\Omega)$	$Z_P(\Omega)$	
50							
200							
500							
1000							
1220							
1500							
5000							
10000							
20000							

AC resistance

Build a plots Z vs. f (experimental and theoretical on one drawing, apply a logarithmic scale for f):

- For series connection of inductance and capacitance.
- For parallel connection *L* and *C*.

Explain your results. Draw <u>conclusion</u>.

### Questions

- 1. What is alternating current? Build plots *I* vs. *t* and *U* vs. *t*. Write down a law of change in voltage and current.
- 2. How are voltage and current related in the resistor (for an alternating current circuit)?
- 3. What is a capacitor? What determines its capacity (equation, unit)? Why can't direct current pass through a capacitor?
- 4. What is an inductor? What determines its inductance (unit)? Why does a self-induction EMF appear in it when the current in the coil changes?
- 5. Which electric circuit is called an *L*-*R*-*C* series circuit? Where is the energy of the electromagnetic field stored in the *L*-*R*-*C* series circuit?

- 6. Explain why harmonic undamped oscillations of charge and current occur in the L-C series circuit. According to which law do the charge of the capacitor and the current in the inductor change with time?
- 7. How does the oscillation period in an L-R-C series circuit depend on the magnitude of the electric capacitance and inductance of the elements?
- 8. Describe the phenomenon of resonance in an *L*-*R*-*C* series circuit. How is the phenomenon of resonance used in radio engineering? Draw a resonance curve for two different resistance values.

# 4. OPTICS

# 41. Centred optical systems

# **Objective**

Studying the image formation in centred optical systems (COS).

# <u>Tasks</u>

Studying the methods of measuring the focal lengths of converging and diverging lenses.

Observation of the dependence of the image on the position of the object in relation to the focus of the lens.

# Geometrical optics

Geometrical optics is a branch of optics where light propagation is described by trajectories called rays. Light rays can be thought as geometrical lines that start at sources, pass through media, and end at detectors; they can be roughly defined as the path along which light energy travels. For a ray passing through a number of pieces of homogeneous media separated by refracting surfaces and possibly undergoing reflections as well, the ray path can also be determined in terms of the laws of reflection and refraction. Snell's law of refraction states that the sine of the angle between the normal and the incident ray bears a constant ratio to the sine of the angle between the normal and the refracted ray; with all three directions being coplanar. Behaviour of light rays (strictly speaking, paraxial rays) passing through a thin converging lens can be described by three main rules: (1) a ray parallel to the optical axis passes through the focus after the lens; (2) a ray passing through the focus becomes parallel to the axis; (3) a ray passing through the centre of the lens does not undergo refraction and remains straight.

# Experimental setup

- An optical bench;
- Incandescent lamp in a case and power supply;
- Aspherical condenser with diaphragm holder;
- Semi-transparent screen;
- Converging lenses with different focal lengths (f = 50, 100, 150, 200, and 300 mm);
- Diverging lens with the focal length of -100 mm;

- Two transparent glasses with a drawing for studying image formation;
- Measuring gauge (2 m).

Task 1. Determining the focal length of a converging thin lens

If the distance between an object and a lens  $(-a_1)$  is changed, then the image is formed at different distances  $a_2$  from the lens according to the equation

$$\frac{1}{-a_1} + \frac{1}{a_2} = \frac{1}{f_2} , \qquad (41.1)$$

where the focal length  $f_2$  (or  $-f_1$ ) of the lens can be found from the plot of  $1/a_2$  vs.  $1/a_1$  as the reciprocal of the segment cut off from the vertical axis.



Fig. 41.1. Ray diagram in a thin lens (left) and experimental setup for determining the focal length of a converging lens (right)

### Algorithm of measurements

- 1. Install the lamp, converging lens, and screen on the optical bench (Fig. 41.1).
- 2. Insert the arrow-shaped diaphragm into the condenser holder.
- 3. Turn on the lamp.
- 4. Move the screen to the end of the bench.
- 5. By moving the lens between the screen and the object (diaphragm), get a well-defined (clear) image of the arrow on the screen.
- 6. Measure the distances  $(-a_1)$  and  $a_2$  (both numbers are positive).
- 7. Reduce the distance between the object and the screen by 3–4 cm and again find the distances  $(-a_1)$  and  $a_2$  that provide a clear image.
- 8. Repeat this 5 times.

9. Build a plot  $1/a_2$  vs.  $1/a_1$  and determine the focal length. Analyse the results.

Task 2. Determining the focal length of converging lenses by the Bessel's method

If the object and the image are separated by a distance greater than 4f, then two positions of the lens can be found, one of which gives a magnified, and the other yields a decreased image (Fig. 41.2 (left)). The distance between the screen and the object in this experiment remains constant.



Fig. 41.2. Ray diagram (left) and experimental setup (right) for determining the focal length of a converging thin lens by the Bessel's method

Let us introduce the values  $S = (-a_1) - (-a_1)$  and  $L = (-a_1) + a_2$ (the meaning of L and S is clear from the sketch in Fig. 41.2). Allowing for Eq. (41.1), we can derive:

$$f = \frac{L^2 - S^2}{4L}.$$
 (41.2)

#### Algorithm of measurements

- 1. Install the lamp with the aspheric condenser at one end of the bench. Insert the glass with a drawing into the diaphragm holder.
- 2. Install the semi-transparent screen about 50 cm away from the object.
- 3. Install the lens with f = 100 mm between the diaphragm and the screen (Fig. 41.2 (right)).

- 4. Move the holder with the lens toward the object (closer to the lamp) until a well-defined image appears on the screen, measure the distance  $a_2$  between the lens and the screen.
- 5. Move the lens toward the screen until you get another clear image. Measure the distance  $a'_2$ , between the lens and the screen.
- 6. Determine  $S = (-a'_1) (-a_1) = a_2 a'_2$ , and calculate the focal length of the lens by Eq. (41.2).
- 7. Repeat steps 3-6 with f = 150 mm lens. Before starting the experiment move the screen so that L > 4f. Analyse the results.

*Task 3*. Determining the focal length of a converging lens by the autocollimation

Auto-collimation method is based on the reversibility of light propagation. If an object is placed at the focus of a lens, then after the lens there will be a light ray parallel to the optical axis. A plane mirror placed beyond the lens reflects this ray so that the object's image will be observed near the object (Fig. 41.3). The distance d between the lens and the image will be equal to the focal length of the lens f.



Fig. 41.3. Experimental setup for determining the focal length by the auto-collimation

### <u>Algorithm of measurements</u>

- 1. Install the lamp with the aspheric condenser on the optical bench as shown in Fig. 41.3.
- 2. Install the lens with f = +150 mm so that light passes through the lens parallel to the optical axis. The distance between the lens and the diaphragm holder should be approximately equal to the focal length of the lens.
- 3. Insert the glass with a drawing (object) and a sheet of white paper (screen for observing the object) into the diaphragm holder according to Fig. 41.3. Both the paper and the object should cover half of the condenser lens.
- 4. Install the mirror beyond the lens. The distance from the lens and the mirror may be less than the focal length.
- 5. Adjust the position of the lens until you get a well-defined image on the paper. It may happen that the position of the mirror and lens will have to be adjusted until the image is the same size as the object.
- 6. Measure the distance *d* between the lens and the object plane.
- 7. Repeat the experiment with other lenses. Analyse the results.

#### Task 4. Determining the focal length of the diverging lens

Focal length of a diverging lens is determined using an additional converging lens. If we place a diverging lens 2 on the path of the rays which are emitted from the source S and converge at point  $S_1$  after being refracted in a converging lens 1, so that the distance  $a_1$  is less than the focal length |f| of this diverging lens, then the image of the source S will move away from the lens 1. Imagine that it moves to the point  $S_2$ . Now  $S_1$  is the object and  $S_2$  is the corresponding image for the lens 2. According to Eq. (41.1), the back focus of a diverging lens can be found as

$$f_2 = \frac{a_1 a_2}{a_1 - a_2} \,. \tag{41.3}$$



Fig. 41.4. Ray diagram (left) and experimental setup (right) for determining the focal length of a diverging lens

### Algorithm of measurements

- 1. Install the lamp, the converging lens with f = 100 mm, and the screen on the optical bench, as shown in Fig. 41.4.
- 2. Insert the glass with the drawing in the holder for the diaphragms.
- 3. Install the lens at a distance of  $\sim$ 30 cm from the object and obtain a welldefined reduced image of the drawing by moving the screen; remember this position of the screen (point *S*<sub>1</sub>).
- 4. Install the studied diverging lens between the converging lens and the screen.
- 5. Find a new sharp image of the object by moving the screen (point  $S_2$ ).
- 6. Having found the distances  $a_1$  and  $a_2$  (Fig. 41.4), calculate the focal length *f* of the diverging lens using the Eq. (41.3). Analyse the results.

*Task 5*. Determining the focal lengths of the converging and diverging lenses using parallel light rays



Fig. 41.5. Initial experimental setup



Fig. 41.6. Experimental setup for determining the focal lengths of converging (left) and diverging (right) lenses using parallel rays

### Algorithm of measurements

- 1. Install the lamp with the aspheric condenser and the semi-transparent screen on the optical bench (with the screen parallel to the bench as shown in Fig. 41.5). The lens is not needed yet.
- 2. Make a parallel light ray going along the optical axis. To do this, get a sharp image of the lamp filament (horizontal) at the most distant point (on a wall) by moving the lamp relative to the condenser.

- 3. Fix a sheet of white paper on the screen using Scotch tape. Make sure that the ray is a parallel ray.
- 4. Install the lens with f = 100 mm in the holder in front of the screen.
- 5. Mark the point of convergence of the refracted light (Fig. 41.6 (left)) and measure the focal length.
- 6. To find the focal length of the diverging lens, fold the paper twice and fix it on the screen so that the fold line coincides with the side of the screen near the lens (Fig. 41.6 (right)).
- 7. Mount the lens with f = -100 mm in the holder in front of the screen.
- 8. Draw the shape of the refracted ray on paper by drawing lines that bound the illuminated area.
- 9. Take the paper off, trace the lines back to the intersection point, and determine the focal length (Fig. 41.6 (right)). Analyse the results.

<u>Analyse the results</u>. Make <u>conclusions</u> corresponding to the objective of the laboratory work.

# Questions

- 1. Converging and diverging lenses. How are they shown in diagrams?
- 2. Thin lens. Equation (deriving).
- 3. Optical power of a lens (equation, unit of measurement).
- 4. Focal length. What factors affect it?
- 5. Centered optical system: definition, characteristics.
- 6. Draw and describe images of an object formed by a thin converging lens in three cases: with an object between the lens and the focus, between focus and double focal length, beyond the double focal length.
- 7. Similarly, draw and describe images made by a diverging lens.
- 8. Optical system of the eye: dimensions, shape, composition.
- 9. Adaptation and accommodation. Optical power of the crystalline lens. The image formed on the retina.
- 10. Myopia and hyperopia, how they are corrected.

# 42. Absorption spectra of tinted glass samples

### <u>Objective</u>

Registration of the absorption spectra of tinted glass samples using a spectrophotometer.

## <u>Tasks</u>

Register the light of an incandescent lamp passing through colored glass using a spectrometer, and compare this light with the continuous spectrum of the lamp light.

Calculate the transmittance and optical density of colored glass.

Light is that part of electromagnetic radiation that is visible to the eye. This radiation is found in the wavelength range of about 380 nm to 780 nm and consists of photons. Violet light with a wavelength of 400 nm is the most energetic radiation in the visible spectrum, and red light with a wavelength of 700 nm is the least energetic.

As light waves propagate through a medium, a decrease in their intensity is observed. This phenomenon is called the absorption of light in matter. Absorption of light is associated with the transformation of the electromagnetic field energy of the wave into other types of energy. According to the electron theory, the interaction of light and matter is reduced to the interaction of the electromagnetic field of the light wave with the atoms and molecules of the matter. Electrons that make up atoms can oscillate under the influence of the alternating electric field of a light wave. Part of the energy of the light wave is spent on the excitation of electron oscillations. Part of the energy of electron oscillations is again converted into the energy of light radiation as well as into other forms of energy, for example, into the energy of thermal motion.

# Bouguer–Lambert law

Let a monochromatic wave of intensity  $I_0$  fall on a flat layer of matter of thickness l (Fig. 42.1). Let us divide this layer into a series of elementary layers with thickness dx (Fig. 42.1). The intensity of the wave approaching the layer lying at depth x is denoted by I(x). The intensity of light dI(x) absorbed by a layer dx is proportional to the light I(x) incident on this layer and the thickness of the layer dx.

Considering that the intensity decreases as you go deeper into the substance:

$$dI = -\alpha I(x)dx , \qquad (42.1)$$

where  $\alpha$  is the absorption coefficient, which depends on the frequency of the incident wave. The minus sign indicates that there is a decrease in intensity, i.e., *dI* is a negative value.



Fig. 42.1. Towards the derivation of Bouguer-Lambert law

To calculate the total absorption of light in a layer of a substance of finite thickness l, we integrate expression 42.1:

$$\int_{I_0}^{I} \frac{dI(x)}{I(x)} = -\int_0^{I} \alpha dx , \qquad (42.2)$$

$$\ln I - \ln I_0 = -\alpha l , \qquad (42.3)$$

$$I = I_0 e^{-\alpha l} \quad , \tag{42.4}$$

where  $I_0$  is the intensity of light falling on a layer of thickness l, and I is the intensity of light emerging from it.

Eq. 42.4 (Bouguer–Lambert law) is valid only for monochromatic light, since the absorption coefficient depends on the wavelength  $\lambda$ .  $\alpha$  – Characterizes the thickness of a layer of matter that weakens the intensity of monochromatic radiation passing through it by *e* times. The unit of measurement of the absorption coefficient in the SI system is m<sup>-1</sup>. Usually, absorption is selective, i.e., light of different wavelengths is absorbed differently. Since the wavelength determines the color of light, rays of different colors are absorbed differently in a given substance. Transparent, uncolored bodies are bodies that provide low absorption of light of all wavelengths related to the range of visible rays. Thus, glass absorbs only about 1% of the visible rays passing through it in a layer 1 cm thick. The same glass strongly absorbs ultraviolet and far infrared rays. Colored transparent bodies are bodies that exhibit selective absorption within the visible rays. For example, glass that weakly absorbs red and orange rays and strongly absorbs green, blue, and violet rays is "red". If white light, which is a mixture of waves of different lengths, falls on such glass, then only the longer waves will pass through it, causing the sensation of a red color, while the shorter waves will be absorbed. When the same glass is illuminated with green or blue light, it will appear "black" because the glass absorbs these rays.



Fig. 42.2. Absorption bands of light by a substance for a certain range of wavelengths

The dependence  $\alpha = f(\lambda)$  is a curve with a series of maxima, which, in turn, represent the absorption band of light by the substance for a certain range of wavelengths (Fig. 42.2). The transmission coefficient is equal to the ratio of the intensity of light *I* passing through a layer of matter to the intensity of light  $I_0$  falling on this layer:

$$\tau = \frac{I}{I_0} , \qquad (42.5)$$

$$\alpha = -\frac{2.3 \cdot lg\tau}{l} \,. \tag{42.6}$$

The optical density of a substance D is equal to the logarithm of the reciprocal of the transmittance:

$$D = lg \frac{l_0}{l} = lg \frac{l}{\tau}.$$
 (42.7)

Thus

$$\alpha = \frac{2.3}{l} \cdot D . \tag{42.6}$$

For different substances, the numerical value of the absorption coefficient  $\alpha$  is different and fluctuates within wide limits. As an example, we give the absorption coefficients of visible rays, which are determined by the following values: (0.01–0.03) cm<sup>-1</sup> values for glass (depending

on the type); 0.001 cm<sup>-1</sup> values for water;  $(2 - 4) \times \cdot 10^{-1}$  cm<sup>-1</sup> values for air (depending on humidity).

The principle of operation of the compact spectrometer

Light enters the entrance slit 1 (Fig. 42.3). It is a narrow slit made in an opaque plate and has a fixed width of 25  $\mu$ m.



Fig. 42.3. Spectrometer diagram

The collimating spherical mirror 2 transforms the diverging light beam into a parallel one and directs it to the reflective diffraction grating 3 (Fig. 42.3). The light, decomposed into a spectrum, is focused onto the surface of the detector 6 (silicon linear CCD detector) using a focusing mirror 4 and a collecting lens 5. This detector contains 650 light-sensitive elements (pixels), the size of which is  $14 \times 200 \,\mu$ m. One element count corresponds to the registration of 75 photons. All elements are arranged along one straight line in such a way that each light-sensitive pixel corresponds to a certain wavelength of light. After analog-to-digital conversion of electrical signals from pixels, the spectrum is transferred to a program on a PC in digital form.

# Experimental setup

- Compact spectrometer;
- Fibre holder;
- Lamp housing with cable;

- Bulb, 6 V/30 W;
- Condenser with diaphragm holder;
- Transformer, 6/12 V;
- Holder with spring clamps;

Various filters:

- Light filter, dark red;
- Light filter, blue-green;
- Light filter, blue with violet;
- Optical bench, S1 profile, 1 m;
- Rider with clamp;
- PC with Windows.



Fig. 42.4. Experimental setup

# Algorithm of measurements

- 1. Place the lamp in the housing, but do not yet make a connection to the 6 V transformer output (Fig. 42.4).
- 2. Initially leave the filter holder empty, i.e. without any filter in the spring clamps.
- 3. Activate  $\Box$  to begin a new measurement.
- 4. Select the Intensity I1 display.
- 5. Start the measurement with  $\blacktriangleright$  .
- 6. Connect the lamp to the 6 V transformer output.
- 7. Align the fibre optic waveguide to maximise intensity. Adapt the integration time, either directly or with  $\Theta$  or  $\oplus$ , such that

the maximum intensity lies between 75 % and 100 %. Do not change the integration time again after this.

- 8. Switch off the light again to record the background spectrum.
- 9. Open the **Offset I0** display. The displayed spectrum will be removed from subsequent measurements as the background spectrum.
- 10. Change to the **Reference I2** display.
- 11. Connect the lamp again to the 6 V transformer output.
- 12. The displayed spectrum serves as a reference spectrum for the following measurement. Suspend reference measurement with ■.
- 13. Place a filter in the spring clamps of the holder.
- 14. The light spectrum passing through the filter can now be seen on the **In-tensity I1** display. The reference spectrum is also displayed in grey.
- 15. The filtered spectrum's relationship to the reference curve is calculated and presented in the **Transmission T** display.
- 16. Optical density (D) will be calculated and presented in the Extinction E display.
- 17. The button "red circle" can be used to save the transmission spectrum for all displays simultaneously.
- 18. Repeat the experiment with other filters.
- 19. Determine the main parameters of the absorption light filters. Insert the results of measurements and calculations into Table 42.1.

*Table 42.1* 

Light	$\lambda$ (nm),	λ (nm),	λ (nm),	τ,	<i>D</i> ,	$\alpha$ (m <sup>-</sup>
filter	(transmission	(transmission	(transmis-	max	max	1)
number	25%)	50%)	sion 75%)			
1						

Results of measurements and calculations

Analyse the results. Make conclusion.

# Questions

1. Draw a schematic diagram of a light wave with its main parameters: wavelength, amplitude, frequency... Specify the units of measurement and the relationships between these parameters.

- 2. What is light?
- 3. Write Bouguer–Lambert law and explain the physical meaning of the absorption coefficient  $\alpha$ .
- 4. What does the absorption coefficient depend on?
- 5. What is the transmission coefficient?
- 6. Name the spectral characteristics of glass.
- 7. Optical spectra: atomic (lines), molecular (bands), continuous. Quantum-mechanical basis of line spectra of atoms.
- 8. Absorption and emission spectra. How are they produced? How do they look like?
- 9. Qualitative and quantitative spectral analysis. What is color? How do people (animals) distinguish different colors?

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