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WEIL FUNCTORS AND PRODUCT PRESERVING FUNCTORS ON THE CATEGORY OF MANIFOLDS DEPENDING ON PARAMETERS

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1. Introduction

Let $\mathbb{A} = \mathbb{R}[[N]]/\mathbb{I}$ be a local Weil algebra of width N and height q, i.e., a quotient algebra of the algebra $\mathbb{R}[[N]]$ of formal power series in N variables modulo an ideal $\mathbb{I}$ ([1]–[3]), and let $\overset{\circ}{\mathbb{A}}$ be the maximal ideal of $\mathbb{A}$ consisting of nilpotent elements. $\mathbb{A}$ can be decomposed into the semidirect sum

$$\mathbb{A} = \mathbb{R} + \overset{\circ}{\mathbb{A}}. \tag{1}$$

The Weil functor $T^{\mathbb{A}} : \mathcal{M}f \to \mathcal{FM}$ [1] defined by a local algebra $\mathbb{A}$ assigns to a smooth manifold M_n the bundle $T^{\mathbb{A}}M_n$ of $\mathbb{A}$-velocities [2] (infinitely near points of $\mathbb{A}$-type [1], $\mathbb{A}$-jets [4]).

The Weil bundle $T^{\mathbb{A}}M_n \to M_n$ of $\mathbb{A}$-velocities over a smooth manifold M_n is defined to be the set of equivalence classes of germs $(\mathbb{R}^N, 0) \xrightarrow{f} M_n$ with respect to the following equivalence relation: germs f and g are equivalent if the Taylor series of mappings $h \circ f$ and $h \circ g$, where h is a chart on M_n, coincide modulo $\mathbb{I}$. By the theorem on $\mathbb{A}$-smooth mappings [3], the coordinate transformations on $T^{\mathbb{A}}M_n$ induced by coordinate transformations $x^{i'} = \varphi^{i'}(x^i)$ on M_n are of the form

$$X^{i'} = x^{i'} + \overset{\circ}{X}{}^{i'} = \sum_{|p|=0}^{q} \frac{1}{p!} \frac{D^{|p|}\varphi^{i'}}{Dx^p} \overset{\circ}{X}{}^p, \tag{2}$$

where $X^i = x^i + \overset{\circ}{X}{}^i$ is the expansion in accordance with (1), and $\varphi^{i'}(x^i)$ are $\mathbb{A}$-valued smooth functions.

The Weil bundle $T^{\mathbb{A}}M_n$ over a smooth manifold M_n defined by a local Weil algebra $\mathbb{A}$ of height q ([1], [2], [5]) is associated with the q-frame bundle B^qM_n whose structure group is the differential group G^q_n. The structure of an $\mathbb{A}$-smooth manifold on $T^{\mathbb{A}}M_n$ ([4], [5]) gives another principal bundle associated with $T^{\mathbb{A}}M_n$, namely, the $\mathbb{A}$-affine frame bundle $B(\mathbb{A})M_n$ whose structure group is the so-called $\mathbb{A}$-affine group $D_n(\mathbb{A})$ [3].

The geometry of Weil bundles and the lifts of various differential-geometrical objects from a manifold M_n to the Weil bundle $T^{\mathbb{A}}M_n$ were studied in [3], [5]–[9] and other papers (see, e.g., the survey [5]).

In [10], we studied the category $\mathcal{M}f^N$ of manifolds depending on N parameters whose objects are trivial bundles $p : M \times \mathbb{R}^N \to \mathbb{R}^N$, where M is a smooth manifold, and morphisms are commutative

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