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Optimality Conditions in a Control Problem for a Linear System With Bilinear Functional

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Abstract—We consider an optimization problem for a linear controlled system with a bilinear-modular functional containing a penalty-type parameter. We formulate sufficient optimality conditions on the base of the notion of a strongly extremal control. The obtained results are presented as inequalities with respect to scalar functions, where the dependence on parameters of the problem is explicit. We adduce some illustrative examples.

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INTRODUCTION

We consider a nonconvex optimal control problem with a bilinear functional and a linear phase system. Based on the maximum principle we establish sufficient optimality conditions. To this end, we apply the notion of a strongly extremal control that maximizes the Pontryagin function on the set of phase or conjugate trajectories [1].

In view of specific properties of the problem under consideration, we can transform optimality conditions with the help of explicit expressions for extremal values of auxiliary problems. As a result, we obtain optimality conditions in the desired form, namely, we state them as inequalities with respect to scalar functions on a certain time interval. The advantage of this approach is the dual character of obtained conditions, namely, they represent pairs of independent symmetric correlations.

Papers [2–6] are relevant to the topic of this work.

1. THE PROBLEM. OPTIMALITY CONDITIONS

Consider the following optimal control problem:

$$\Phi(u) \to \max, \quad u \in V, \tag{1}$$

where

$$\Phi(u) = \langle c, x(t_1)\rangle + \int_{t_0}^{t_1} (\langle a(t), x(t)\rangle u(t) - \gamma|u(t)|)dt,$$

$$\dot{x} = A(t)x + b(t)u, \quad x(t_0) = x^0,$$

$$V = \{u(\cdot) \in PC([t_0, t_1]) : |u(t)| \le 1, \ t \in [t_0, t_1]\}.$$

Here $u(t) \in R$ is a control, $x(t) \in R^n$ is a phase state, $PC([t_0, t_1])$ is the set of piecewise continuous functions, c, x^0, and $\gamma > 0$ are problem parameters.

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