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NUMERICAL SOLVING PROBLEMS OF IDENTIFICATION OF RIGHT SIDE OF PARABOLIC EQUATION

P.N. Vabishchevich

In the theory of inverse problems of heat exchange (see [1]–[3]) the problems of restoration of unknown heat source by additional measurement of temperature are considered. In a similar way one can formulate some important applied problems of the hydrogeology.

The cases where the solution is known in a small number of points (see [4]–[6]) can be singled out as autonomous problems. In these conditions for the uniqueness of the solution it is necessary to restrict the class of right sides of parabolic equations. In many applied problems it is intrinsic to assume that the unknown is the right side as a function of time.

The mentioned inverse problems in a series of important cases can be conveniently formulated as problems of control (see [7]). For an approximate restoration of the unknown right side different approaches are used, based first of all on methods of regularization (see [8].) Such a general algorithm was applied, for example, in [9], [10] to solving essentially ill-posed equations of identification for multidimensional parabolic equations.

In this article we consider the inverse problem of restoration of the right side of one-dimensional parabolic equation by a known solution. We note specific features of this problem, related to its evolutionary character, the possibility of a consecutive determination of the right side within increase of the time. The process of numerical solving is carried out with the use of gradient iteration methods of minimization of the discrepancy functional (see [11], [12]). A regularization in the iteration methods of solving some ill-posed problems is attained at the expense of coordination of the number of iterations and the error of input data (see [12]–[14]). We represent results of numerical solving the model problem, which demonstrate the possibilities of identification of the right side of the parabolic equation under various levels of errors in the input data.

1. *Model problem.* In the capacity of a model problem we consider the problem of restoration of the right side of a one-dimensional parabolic equation. Let us start with the statement of the direct problem. Solutions $u(x,t)$ are defined in the rectangle

$$\overline{Q}_T = \overline{\Omega} \times [0,T], \quad \overline{\Omega} = \{x \mid 0 \le x \le l\}, \quad 0 \le t \le T.$$

The function $u(x,t)$ satisfies the equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial u}{\partial x}\right) + f(x,t), \quad 0 < x < l, \quad 0 < t \le T, \tag{1}$$

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