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Sets Convex in a Cone of Directions

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Abstract—We consider sets which are convex in directions from some cone K. We generalize some well-known properties of ordinary convex sets for the case of K-convex sets and give some applications in optimization theory.

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1. Introduction. In this paper we propose a generalization of the notion of convexity. The definition of a new class of sets, as usually (see, for example, surveys [1–3]), is based on the known properties convex structures. We require its fulfillment only on a certain set given in advance (but not everywhere). In this case, the main property is that the segment connected two points of a set belongs to this set. We consider not all segments but only those that are parallel to given directions. In the case of one direction, we get the well-known class of sets convex in only one direction [4].

The convexity in several directions (in a sector of directions) was first described in [5] as a property of a certain class of domains; it appeared to be a key property in economic problems. The study of sets and functions convex in a cone of directions has allowed one to take into account such effects as the external diseconomy, the altering relation to the consumer risk, and so on [6–8]. The further investigations lead to the necessity of obtaining analogs of known properties of usual convex sets; this is the goal of the present paper.

2. The main results. Let K be a convex cone in R^n. Let us give a definition of a K-convex set.

Definition 1. A set X in R^n is said to be *K-convex*, if conditions $x, y \in X, \pm(x-y) \in K$ imply that $[x, y] \subset X$.

Here the notation $\pm(x-y) \in K$ means that at least one of vectors $(x-y)$ and $(y-x)$ belongs to K, and $[x, y]$ is the segment connecting points x and y.

Properties of K-convex sets and usual convex ones are similar. Let us describe some of them (one can prove them by modifying known proofs for the case of the usual convexity).

Proposition 1. *If X is a K-convex set, then it contains all convex linear combinations of any finite number of its points such that the difference d between any two chosen points satisfies the condition $\pm d \in K$.*

Proposition 2. *The intersection of sets $X_\lambda (\lambda \in \Lambda)$, which are convex in a cone of directions K is K-convex.*

Proposition 3. *Let G be a K-convex set in R^n. Assume that in K there exist $(n-1)$ linear independent vectors a^i, $i = 1, 2, \dots, n-1$. Then the section of the set G by a hyperplane parallel to the minimal subspace containing vectors a^i, $i = 1, 2, \dots, n-1$ is a convex set.*

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