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Non-Contradictory Aggregation of Strict Order Relations

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Abstract—We study the problem of collective choice. The profile of individual preferences of experts is defined by relations of strict order. A non-contradictory aggregate preference relation is based on the weighted majority graph that characterizes the degree of superiority of one alternative over another. The aggregate relation also defines a strict order and satisfies requirements to group decisions, namely, the monotony, the preservation of the Pareto relation, the minimality of the distance to expert preferences.

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INTRODUCTION

In almost all areas of human activity, one encounters problems whose solution is very difficult. A decision maker can often fail in finding an optimal solution intuitively due to a large amount of initial information. So there occurs a necessity in the development of mathematically substantiated methods and algorithms for supporting decision-making processes.

In this paper, we consider the problem of collective choice in decision-making processes. There are many papers devoted to this subject; the most known one is the work by K. Arrow [1] where he proves the impossibility of the collective choice. The development of compromise algorithms and the attraction of highly qualified experts in the mentioned research area can improve the solution quality. In this paper, we continue the work of B. G. Mirkin [2], V. I. Vol'skii [3], V. V. Podinovskii [4], and F. T. Aleskerov [5].

We propose a method for aggregating expert preferences defined by strict order relations, in particular, by strict ranking. For aggregating preferences we use the classical majority graph [2, 3] whose edges are endowed with weights that characterize the degree of superiority of some alternative over some other one. The summary relation has to be non-contradictive and consistent with the expert information. The main advantage of the majority graph is the fact that it most fully takes into account opinions of experts. Namely, the relation that corresponds to the majority graph has the minimal total distance to expert preferences. However, the classical majority graph does not satisfy the transitivity condition, in particular, it can contain contours. This makes the choice of the best alternatives more difficult and makes their ranking impossible. The well-known ranking methods [2, 3, 6] do not guarantee the correspondence to the majority graph.

In this paper, we propose an algorithm which allows to eliminate contradictions by destroying contours in the majority graph, maximally preserving its structure, and then to define the aggregate relation as the minimal transitive one that contains a contourless graph. The algorithm also uses the information on the number of experts that prefer some alternative to some other one.

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