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AN ANALYTIC SOLUTION OF A SYSTEM OF DIFFERENTIAL EQUATIONS, DESCRIBING AN AXIALLY SYMMETRIC DEFORMATION OF ORTHOTROPIC SHALLOW SPHERICAL SEGMENT

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A precise solution of the Timoshenko type equations is constructed for an orthotropic spherical segment under a load with the principal vector **P**, which is uniformly distributed along the parallel $\alpha = \alpha_0$ and directed normally to the middle surface of a shell. In doing so, we employ the mathematical analogy to the solution of a similar problem in the Kirchhoff–Love theory (see [1]).

The system of equilibrium equations in terms of stresses and moments takes the following form (see [2], p. 68)

$$\frac{1}{\alpha}\frac{d}{d\alpha}(\alpha T_\alpha) - \frac{1}{\alpha}T_\beta = 0, \quad \frac{1}{\alpha}\frac{d}{d\alpha}(\alpha M_\alpha) - \frac{1}{\alpha}M_\beta - RN_\alpha = 0, \quad \frac{1}{\alpha}\frac{d}{d\alpha}(\alpha N_\alpha) - (T_\alpha + T_\beta) + RZ = 0. \tag{1}$$

By introducing the stress function φ so that

$$T_\alpha = \frac{1}{R^2\alpha}\frac{d\varphi}{d\alpha}, \quad T_\beta = \frac{1}{R^2}\frac{d^2\varphi}{d\alpha^2}, \tag{2}$$

and using the relations for the moments and the cross-cutting force (see [2], pp. 35, 38)

$$M_\alpha = \frac{D_\alpha}{R^2}\left(\frac{d^2}{d\alpha^2} + \frac{v_\beta}{\alpha}\frac{d}{d\alpha}\right)(\Phi - w), \quad M_\beta = \frac{D_\beta}{R^2}\left(\frac{1}{\alpha}\frac{d}{d\alpha} + v_\alpha\frac{d^2}{d\alpha^2}\right)(\Phi - w), \quad N_\alpha = \frac{c^2}{R}\frac{d\Phi}{d\alpha}, \tag{3}$$

as well as the contingency conditions for the strain, we obtain the system of the solving equations

$$D_\alpha L(\Phi - w) - R\nabla^2\varphi + R^4 Z = 0, \quad L(\varphi) - RHE_\beta\nabla^2 w = 0, \quad \nabla^2\left(\Phi - \frac{1}{c^2R}\varphi\right) + \frac{R^2Z}{c^2} = 0. \tag{4}$$

Here R and H are the sphere radius and the thickness of the segment, respectively; Z is the intensity of the transverse load which has the principal vector **P** and is distributed uniformly along the parallel $\alpha = \alpha_0$; E_α and E_β are the elasticity moduli of the material in the meridional and parallel directions, respectively; v_α and v_β are the Poisson coefficients, G stands for the transverse shear modulus, w is the lag of the shell, Φ means the potential part of the transverse shear function (the vortex component vanishes for the axially symmetric strain),

$$\begin{gathered} L \equiv \frac{d^4}{d\alpha^4} + \frac{2}{\alpha}\frac{d^3}{d\alpha^3} - \frac{q^2}{\alpha^2}\frac{d^2}{d\alpha^2} + \frac{q^2}{\alpha^3}\frac{d}{d\alpha}, \quad \nabla^2 \equiv \frac{d^2}{d\alpha^2} + \frac{1}{\alpha}\frac{d}{d\alpha}, \\ q^2 = \frac{E_\beta}{E_\alpha}, \quad v_\beta = v_\alpha q^2, \quad D_\alpha = \frac{E_\alpha H^3}{12(1 - v_\alpha v_\beta)}, \quad D_\beta = D_\alpha q^2, \quad c^2 = \frac{5}{6}GH. \end{gathered} \tag{5}$$

The main assertion of the present article is the following