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APPROXIMATE SOLUTIONS IN THE PROBLEM OF ASYMPTOTIC ATTAINABILITY

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We investigate the problem of constructing an attraction set (A. S.) subject to asymptotic constraints. We study the presentations of the set of its approximate solutions (AS) defined in terms of filters, ultrafilters, and finite-additive $(0,1)$-measures on measurable spaces (MS) with semi-algebras of sets. We consider the compactifiable case of the problem. For this case we prove that in classes of ultrafilters and finite-additive $(0,1)$-measures the set of all AS (which satisfy the asymptotic constrains) consists only of AS which form the points of the A. S. in the space of estimates.

1. Introduction

Consider the set $E \neq \emptyset$, the topological space (TS) $(\mathbf{H}, \theta)$, the mapping $\mathbf{h} : E \longrightarrow \mathbf{H}$, and the nonempty family $\mathcal{E}$ of subsets of the set E. We study the asymptotic construction conceptually connected with the choice of directivities (e_α) in E such that for any $U \in \mathcal{E}$, beginning with a certain moment, $e_\alpha \in U$ ([1], Chap. 2). We call these directivities AS. Among the latter we naturally distinguish AS (e_α) such that $(\mathbf{h}(e_\alpha))$ is a directivity (in $\mathbf{H}$) which converges in the TS $(\mathbf{H}, \theta)$. We say that these AS (e_α) form the A. S. The possible existence of other AS which satisfy in the mentioned sense the $\mathcal{E}$-constraint and at the same time do not form any point of the A.S. (even under an arbitrary "thinning" up to a subdirectivity) is actually shown in ([2], § 7.2). The aim of this paper is to introduce the corresponding sets of AS. It is difficult if for the formalization of the mentioned AS one uses the directivities themselves. A natural analogue of a directivity is a filter, in particular, the use of an ultrafilter is interesting. However, the constructive definition of the latter is very difficult if we mean the ultrafilter of the family of all subsets of E. Evidently, it makes sense to consider an ultrafilter of the MS, whose "unit" is E. It is well known that for some types of MS with semi-algebras and algebras of sets one can describe the corresponding set of ultrafilters completely. The reduction of AS to an ultrafilter of an MS can be presented as a reduction of these AS (initially treated as directivities) to $(0,1)$-measures on this MS. These measures should be, generally speaking, finite-additive. This fact is essential even in rather simple cases of constructing extensions. The mentioned reduction needs certain restrictions on the used MS. However, one can always satisfy them, using all possible subsets of E as measurable ones, though it is undesirable.

2. Common notation and definitions

In what follows, we understand a family as a set such that all its elements are also some sets and accept the axiom of choice. We denote by $\mathcal{P}(X)$ (by $\mathcal{P}'(X)$) the family of all (all nonempty) subsets of the set X. For arbitrary sets A and B we denote by B^A ([3], § II.6) the set of all functions

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