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Some Cases of Efficient Factorization of Second-Order Matrix Functions

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Abstract—We obtain sufficient conditions for the factorization of second-order matrix functions in the closed form.

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Let Γ be a simple smooth closed contour in the complex plane dividing the latter into two domains D^+ and D^- ($\infty \in D^-$). We understand a factorization of an H-continuous on Γ matrix function

$$G(t) = \begin{pmatrix} g_{11}(t) & g_{12}(t) \\ g_{21}(t) & g_{22}(t) \end{pmatrix}, \quad \Delta(t) = \det G(t) \neq 0, \quad t \in \Gamma, \tag{1}$$

as its representation in the form $G(t) = G^+(t)G^-(t)$, $t \in \Gamma$. Here $G^\pm(t)$ are limit values (in the corresponding domains) of some matrix function $G(z)$ whose elements are piecewise analytic and can have a polar singularity at infinity, $\det G(z) \neq 0$ in a finite part of the plane, and the order of $\det G^-(z)$ equals the sum of orders $\varkappa_1$ and $\varkappa_2$ of rows of the matrix function $G^-(z)$ at infinity. These numbers are called partial indices and their sum $\varkappa = \operatorname{ind} \det G(t)$ is called the total index of the matrix function $G(t)$. If the sum of orders of rows of $\det G^-(z)$ at infinity exceeds the order of the determinant itself at infinity, then such a representation of the matrix function $G(t)$ is said to be normal, because the matrix function

$$X(z) = \{G^+(z),\ z \in D^+;\ [G^-(z)]^{-1},\ z \in D^-\}$$

is a normal matrix corresponding to a homogeneous linear conjugation problem and by means of a known algebraic algorithm can be reduced to a canonical matrix ([1], pp. 30, 40). Finally, if at a certain finite point $z_0 \in D^+$ (D^-) $\det G^+(z_0) = 0$ $(\det G^-(z_0) = 0)$, then by using the method for constructing the normal matrix described in [2] we get the normal representation for $G(t)$ allowing us to efficiently construct a factorization in this case as well. In [3] one shows that the existence of a piecewise meromorphic solution to the linear conjugation problem allows one to construct the canonical matrix without constructing the normal one. The factorization problem for meromorphic matrix functions is considered in [4]. In [5] one proposes a constructive approach to the factorization problem for meromorphic second-order matrix functions. In [6] one studies a special class of second-order matrix functions which are reducible to the triangular form by the multiplication by rational matrices from the left and from the right. In [7] one continues this study and classifies the second-order matrix functions that allow such transforms with respect to the number of linearly independent elements of matrix functions over the field of rational functions.

In [8] we consider the characteristic system of singular integral equations

$$A\mathbf{w} + BS[K\mathbf{w}] = \mathbf{f}, \tag{2}$$

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