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Cancellable Elements of the Lattice of Epigroup Varieties

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Abstract—We completely describe all commutative epigroup varieties that are cancellable elements of the lattice **EPI** of all epigroup varieties. In particular, we prove that a commutative epigroup variety is a cancellable element of the lattice **EPI** if and only if it is a modular element of this lattice.

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1. INTRODUCTION

An *epigroup* is a semigroup S in which some power of each element is a *group element*, i.e., belongs to some subgroup in S. Extensive information on epigroups can be found in [1, 2]. The class of epigroups is rather wide. It includes, in particular, all *periodic* semigroups (in which some power of each element is an idempotent) and all *completely regular* semigroups (each element of which is a group element).

It is natural to consider epigroups as *unary semigroups*, i.e., semigroups with additional unary operation defined as follows. Let S be an epigroup. Let e be an idempotent from S. Denote by G_e the maximal subgroup in S for which e is the identity and by K_e the set of all elements of S some power of which belongs to G_e. By the definition of an epigroup, for every element $x \in S$, there exists an idempotent x^ω such that $x \in K_{x^\omega}$. It is known (see, e.g., [1, 2]) that the idempotent x^ω is defined uniquely and $xx^\omega = x^\omega x \in G_{x^\omega}$. Denote by $\overline{x}$ the inverse element of xx^ω in the group G_{x^ω}. The mapping $x \longmapsto \overline{x}$ is exactly the unary operation on an epigroup S mentioned above. It is called the *pseudoinversion*. An element $\overline{x}$ is called the *pseudoinverse* of x. Everywhere in the sequel, speaking about epigroups, we will consider them as algebras over the signature consisting of the operations of multiplication and pseudoinversion. This allows us, in particular, to speak about the varieties of epigroups as algebras over the indicated signature. It is known (see, e.g., [1, 2]) that the operation of pseudoinversion in every periodic epigroup can be expressed in terms of multiplication. Thus, periodic varieties of epigroups can be identified with periodic varieties of semigroups.

In recent years, there appear many papers devoted to the study of special elements of various types in the lattice **SEM** of all semigroup varieties and its sublattices. In a number of cases, complete descriptions of such varieties were obtained. The first results of this sort were reflected in [3], § 14, a more complete exposition was presented in [4]. In recent time, in papers [5–7], analogs of a series of these results were obtained for the lattice **EPI** of all epigroup varieties. In this paper, one more result of this sort is presented.

In the lattice theory, special elements of various types are considered. Recall the definitions of those types which are considered in this paper. An element x of a lattice $\langle L; \vee, \wedge\rangle$ is called *neutral* if $(\forall y, z \in L)\ \ (x \vee y) \wedge (y \vee z) \wedge (z \vee x) = (x \wedge y) \vee (y \wedge z) \vee (z \wedge x)$. A neutral element can also be defined as an element which generates together with any two elements of the lattice its distributive sublattice ([8], theorem 254). An element $x \in L$ is called

modular, if $\quad (\forall y, z \in L) \quad y \le z \longrightarrow (x \vee y) \wedge z = (x \wedge z) \vee y,$

cancellable if $\quad (\forall y, z \in L) \quad x \vee y = x \vee z \ \&\ x \wedge y = x \wedge z \longrightarrow y = z.$

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