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RIESZ INTEGRABILITY OF SPECTRAL EXPANSIONS FOR FINITE-DIMENSIONAL PERTURBATIONS OF ONE CLASS OF INTEGRAL OPERATORS

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In the space $L[0,1]$ we consider the operator

$$Af = A_0 f + \sum_{k=1}^{m} g_k(x)(f, v_k), \tag{1}$$

where $A_0 f = \alpha_1 \int\limits_0^x f(t)dt + \alpha_2 \int\limits_0^{1-x} f(t)dt$, $(f, v_k) = \int\limits_0^1 f(t)v_k(t)\,dt$, $g_k(x), v_k(x) \in C^1[0,1]$, $x \in [0,1]$. Suppose that the systems $\{g_k'(x)\}_1^m$, $\{v_k(x)\}_1^m$ are linearly independent and $\delta = \alpha_1^2 - \alpha_2^2 \neq 0$, $\alpha_2 \neq 0$.

We denote by $R_\lambda f = (E - \lambda A)^{-1} A f = \int\limits_0^1 G(x,t,\lambda) f(t)\,dt$ (E stands for the unit operator) the Fredholm resolvent of the operator A. In this article, under some assumptions concerning operator of the form (1), the necessary and sufficient conditions imposed upon the function $f(x)$ are found ensuring the uniform convergence to this function on the whole segment $[0,1]$ of the means of the form

$$-\frac{1}{2\pi i} \int_{|\lambda|=r} g(\lambda, r) R_\lambda f \, d\lambda,$$

where $g(\lambda, r)$ satisfies the following conditions:

a) $g(\lambda, r)$ is continuous with respect to λ in the disk $|\lambda| \le r$ and analytic with respect to λ in the disk $|\lambda| < r$ for any $r > 0$;
b) a constant $C > 0$ exists such that $|g(\lambda, r)| \le C$ for all $r > 0$ and $|\lambda| \le r$;
c) positive numbers β_1, β_2 exist such that

$$g(re^{i\varphi}, r) = O\left(\left| \varphi + \alpha - \frac{\pi}{2} \right|^{\beta_1} \left| \varphi + \alpha + \frac{\pi}{2} \right|^{\beta_2} \right),$$

where $\alpha = \arg \sqrt{\delta}$ (the estimates are uniform with respect to r);
d) $g(\lambda, r) \to 1$ as $r \to \infty$ and for a fixed λ.

Examples of such functions are supplied by functions of the form

$$g(\lambda, r) = g_1(\lambda, r) g_2(\lambda, r),$$

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