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OPTIMIZATION OF DIRECT AND PROJECTION METHODS FOR SOLVING WEAKLY SINGULAR INTEGRAL EQUATIONS

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By the present time the problem of optimization with respect to the exactness order of both direct and projection methods (see [1], [2]) was solved for a series of classes of regular and singular integral equations. In this article the problem is solved for sufficiently large class of weakly singular Fredholm and Volterra equations of the second kind.

Let us consider the class $\mathcal{E}$ of uniquely resolvable second kind integral equations with weak singularity

$$Kx \equiv x(t) + \int_a^{\sigma(t)} \mu(|t-s|)h(t,s)x(s)\,ds = y(t), \quad -\infty < a \le t \le b < \infty, \tag{1}$$

where $\sigma(t) = t$ or $\sigma(t) \equiv b$, and $\mu(\tau)$ is a fixed function for the whole class $\mathcal{E}$. With respect to $\mu(\tau)$ we will assume that it possesses an *arbitrary integrable singularity* only at the point $\tau = 0$.

Let the class $\mathcal{E} = \mathcal{E}_1$ be given by the relations [1]

$$h \in H_{\omega_1}(A_1;[a,b]) \text{ with respect to the variable } t \text{ uniformly with respect to } s, \tag{2}$$

$$y \in H_{\omega_2}(A_2;[a,b]), \tag{3}$$

where ω_1, ω_2 are given continuity modules, while $A_1, A_2, \dots$ are some absolute positive constants.

1. *Structure properties of weakly singular integral operator.* We denote by $X^* = \{x^*\}$ a set of solutions of equations (1) from the class $\mathcal{E}$. Note that in the class X^* all elements are bounded with respect to the norm of the space $C[a,b]$ by a certain constant A_3 depending, generally speaking, on characteristics of the class $\mathcal{E}$.

Let Z be a set of functions

$$z(t) = \int_a^{\sigma(t)} \mu(|t-s|)h(t,s)x(s)ds, \tag{4}$$

when $h(t,s)$ runs over a class defined by relation (2), while $x(s)$ the class X^*. We introduce a function

$$\omega_3(\delta) = \left\{ \int_0^\delta |\mu(\tau)|\,d\tau, \quad 0 < \delta \le \tau_0; \quad \int_0^{\tau_0} |\mu(\tau)|\,d\tau, \quad \tau_0 < \delta \le b-a \right\},$$

where τ_0 is the closest to zero point such that $\mu(\tau)$ is monotone and has same sign in the half-segment $(0,\tau_0]$. Obviously, the function $\omega_3(\delta)$, $0 < \delta \le b-a$, is a continuity module.

[1] Here we denote by $H_\omega(A;[a,b])$ the generalized Hölder class (see, e.g., [3], [4]).