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# Computational Tricks in Linear Programming Without the Use of Artificial Variables

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We describe two computational tricks without the use of artificial variables, concerning the solution of the problem

$$c^T x - \min, \quad Ax = P_0, \quad x \geq 0, \tag{1}$$

where $c = (c_1, c_2, \dots, c_n)^T$, $x = (\xi_1, \xi_2, \dots, \xi_n)^T$, $A$ is a matrix of the dimension $m \times n$ with the rank $m$. Assume that $m < n$. We denote columns of the matrix $A$ by $P_j$, $j = 1, \dots, n$. If $P_1, P_2, \dots, P_m$ is a basis in $R_m$, then we say that a submatrix $B = (P_1, P_2, \dots, P_m)$ of the matrix $A$ is a basis matrix and treat the collection of vectors $P_1, P_2, \dots, P_m$ as a basis $B$. Denote $c_B = (c_1, c_2, \dots, c_m)^T$, $\Delta_j = c_B^T B^{-1} P_j - c_j$, $j = 0, 1, \dots, n$, $c_0 = 0$. As usual, we understand the inequality $x \geq 0$ as the condition $\xi_j \geq 0$ for all $j = 1, \dots, n$. We also use the standard definitions of [1], [2].

One can easy verify that the vector $B^{-1}P_j$ represents a collection of expansion coefficients of the vector $P_j$ in the basis $B$. Components of the vector $B^{-1}P_j$ are said to be basis coordinates of the vector $P_j$. Note that the equality $Ax = P_0$ admits the form

$$\sum_{j=1}^{n} \xi_j P_j = P_0, \tag{2}$$

and the main part of the simplex table represents the extended matrix of the system of linear algebraic equations

$$\sum_{j=1}^{n} \xi_j B^{-1} P_j = B^{-1} P_0, \tag{3}$$

which is equivalent to system (2). In other words, columns of any simplex table which corresponds to a certain basis $B$ represent collections of expansion coefficients of vectors $P_j$ in this basis.

It is well-known (e.g., [1], P. 37) that if a basis matrix $B$ concurrently satisfies the conditions

$$B^{-1} P_0 \geq 0 \tag{4}$$

and

$$\Delta_j \leq 0 \ \forall j = 1, \dots, n, \tag{5}$$

then problem (1) is solved, because coordinates of the vector $B^{-1}P_0$ are basis coordinates of the optimal plan of problem (1), and $\Delta_0$ is the optimal value of the linear form $c^T x$. If a basis $B$ is such that condition (4) is true and (5) is false, then we solve problem (1) by the simplex method. But if condition (5) is true and (4) is false, then we use the dual simplex method.

With an arbitrary initial basis $B$ usually neither condition (4) nor (5) is fulfilled. For such a case in this paper we propose two solution algorithms for problem (1); they use the simplex and dual simplex methods sequentially.

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