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Dirichlet Problem for Third-Order Hyperbolic Equations

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Abstract—We consider Dirichlet problem for third-order linear hyperbolic equations. We prove the existence and uniqueness of classical solution by means of an energy inequality and Riemann's method. We reveal the influence of coefficients at lower derivatives on the well-posedness of the Dirichlet problem.

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Introduction. In the domain $D = \{(x, y) : 0 < x < l, 0 < y < h\}$ we consider the equation

$$Mu \equiv \left(\alpha\frac{\partial}{\partial x} + \beta\frac{\partial}{\partial y}\right)u_{xy} + Lu = g(x, y), \tag{1}$$

where α and β are constant values, $\alpha^2 + \beta^2 \neq 0$, and L is linear differential expression

$$Lu \equiv a(x, y)u_{xx} + 2b(x, y)u_{xy} + c(x, y)u_{yy} + d(x, y)u_x + e(x, y)u_y + f(x, y)u,$$

which refers to one of canonical forms from [1].

Note that hyperbolic equations of the third and higher orders with dominated lower terms are frequently called pseudo-parabolic. Equation (1) is a union in one formula of two variants of generalized pseudo-parabolic Aller equation. Its special cases are studied, for instance, in [2] (pp. 254–256), [3] (pp. 132–138), and [4]–[8].

The families of characteristics of Eq. (1) are real and distinct. This fact substantially affects both on well-posedness of the problems, and on their solvability.

Note that the Dirichlet problem is studied mainly for second-order equations. It is one of main problems of mathematical physics (see, e.g., [9–10]). In [11] it is studied for equations of higher orders, too.

We consider without loss of generality that $\alpha > 0$ and $\beta > 0$. Indeed, if $\alpha < 0, \beta > 0$ or $\alpha > 0, \beta < 0$, then the changes of independent variable $x = 1 - \xi$ or $y = 1 - \eta$ reduce these cases to the case $\alpha > 0$ and $\beta > 0$.

1. Formulation of the problem and uniqueness theorem. In the present paper we study the following Dirichlet problem for Eq. (1): *Find in the domain D a solution to Eq.* (1) *in the class $C^{2,1}(D) \cap C^{1,2}(D) \cap C^{1,1}(\overline{D})$ satisfying conditions*

$$u(0, y) = \varphi_1(y), \quad u(l, y) = \varphi_2(y), \ 0 \le y \le h, \tag{2}$$

$$u(x, 0) = \psi_1(x), \ u(x, h) = \psi_2(x), \ 0 \le x \le l; \tag{3}$$

here $\varphi_i(y), \psi_i(x), i = 1, 2$, are given functions satisfying consistency conditions $\varphi_1(0) = \psi_1(0), \varphi_1(h) = \psi_2(0), \varphi_2(0) = \psi_1(l), \varphi_2(h) = \psi_2(l)$.

We denote by $C^{k,l}(D)$ the class of functions $u(x, y)$, which are continuous together with their partial derivatives $\partial^{m+n}u(x, y)/\partial x^m\partial y^n$ for all $m = 0, 1, \ldots, k, n = 0, 1, \ldots, l$.

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