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THEORY OF DISCONTINUOUS SOLUTIONS OF VARIATIONAL PROBLEMS IN THE CLASS OF GENERALIZED CURVES

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In [1]–[3] the problem about determination

$$\inf I[y] = \inf \int_a^b F(x, y, y')dx \tag{1}$$

was studied in the class U of functions $y(x)$ possessing a finite number of discontinuity points of "wall" type. The suggested method of investigation of the variation problem (1) was based on the construction of extensions of the class U (introduction of the classes U_0 of (y, z)-lines and $\overline{U}_0$ of (y, z)-objects), the additional determination of the functional $I[y]$ on elements of these extensions, and the search of absolute minimals in either the class U_0 or $\overline{U}_0$, which ensures the determination of the approximate minimizing solutions of the class U.

In this article the study of variational problem (1) in the class of essentially discontinuous functions is based on the theory of the generalized Young–McShane curves (see [4]–[6]). The suggested method of investigation enables us to obtain the theorem of existence of a generalized solution of the conjugate parametric problem; what is more, in contrast to [1]–[3] it allows us to prove the existence of the absolute minimum of the variational problem (1) in the initial class of essentially discontinuous functions. In addition, in contrast to [7], [8] this method allows us to weaken the requirements imposed on the smoothness of the integrand F and relieve of the necessity to establish the fact of semicontinuity of the conjugate functional $J[C]$ in the class of absolutely continuous curves possessing at most countable quantity of vertical segments.

1. Admissible class of curves (class Π). By a parametric representation of an absolutely continuous curve C on the plane (x, y) we shall call an absolutely continuous mapping $f(t) = (x(t), y(t))$ of the segment $[t_1, t_2] \subset \mathbf{R}^1$ to $\mathbf{R}^2$. If $t = \varphi(\tau)$ is an absolutely continuous, strictly increasing in $[\tau_1, \tau_2]$ function such that $\varphi(\tau_i) = t_i$ $(i = 1, 2)$, then the parametric representation $f[\varphi(\tau)]$, $\tau \in [\tau_1, \tau_2]$, is assumed to be equivalent to the parametric representation $f(t)$, $t \in [t_1, t_2]$. By a curve $C : f(t)$, $t \in [t_1, t_2]$, we call a class of all parametric representations equivalent to $f(t)$, $t \in [t_1, t_2]$. By the definition, the support $(\{C\} \equiv \{f(t)\})$ of the curve C is a set of values of the function $f(t)$, $t \in [t_1, t_2]$, and, obviously, it does not depend on the form of a parametric representation of the curve.

A rectifiable curve C, which lies in the domain $\Omega = \{(x, y) : x \in [a, b],\ |y| \le y^0\}$ of the plane (x, y), connecting the points $A = (a, a_1)$, $B = (b, b_1)$ $(A, B \in \Omega)$, is related to the class Π if C possesses an absolutely continuous representation $\{f(t) = (x(t), y(t)),\ \dot{x}(t) \ge 0,\ t \in [t_1, t_2]\}$ and therefore its support possesses at most countable number of the segments d_i which are parallel to the axis $0y$.

2. Admissible class of functions (class НΠ). Let C be a curve of the class Π. We denote by P' the projection of any point $P \in \{C\}$ of the plane (x, y) onto the axis $0x$. To each point P' from the segment $[a, b]$ we put into correspondence a set of all $P \in \{C\}$, which possess P'