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HOLOMORPHIC CONFORMAL SUBMERSIONS

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Introduction

The present article is devoted to the study of Kähler manifolds admitting a holomorphic conformal submersion ν with vertical conformality exponent (see Definition 1.1) and totally geodesic leaves. Such manifolds can be treated as Kählerian analogs of crossed products of the Riemannian manifolds. In Examples 1.1 and 1.2, we construct a family of the irreducible Kähler manifolds admitting submersions described above.

The main objective of the present article is to give a description of a local structure of metrics of the Kähler manifolds admitting a submersion ν of the type indicated above. For the submersions with one-dimensional leaves such a description is obtained in Theorem 2.2. It can be shown that, locally, the metrics of the Kählerian y-prolongations in Theorem 2.2 are the Calabi metrics (see [1], (3.2)).

The manifolds, functions, and tensor fields under consideration are supposed to be smooth (of class C^∞). We use the standard definitions and notation, which are adopted, for example, in [2]–[4].

1. Definitions and local examples

Let ν be a submersion of a Riemannian manifold E onto a Riemannian manifold M. At each point $\Xi \in E$ we have the direct orthogonal decomposition of the tangent space $T_\Xi E = V_\Xi + H_\Xi$ into the vertical and horizontal subspaces. The symbols V and H will also stand for the operators of orthogonal projection onto the corresponding distributions. The operators V and H can be extended in a standard way ([2], Chap. 1, proposition 2.12) to the tensor algebra at each point of E. A tensor K (in particular, a vector or a form) will be called vertical (respectively, horizontal) if its horizontal (respectively, vertical) component is zero: $HK = 0$ (respectively, $VK = 0$).

Definition 1.1. Let L and M be the Riemannian manifolds with the metric tensors g and g_M, respectively. A submersion ν from L onto M is said to be conformal if a function f on L exists such that $Hg = \exp(2f)\nu^* g_M$.

The function f is called *the vertical conformality exponent* of the submersion if f is a vertical function, i. e., $Xf = 0$ for every horizontal vector X.

Example 1.1. Let a Kähler 2-form Φ_M given on M possess a Kähler potential F_M, i. e., $\Phi_M = -2i\partial\overline{\partial}F_M$, and let $y(t)$ be a real function on a real interval (a, b) (the values $a = -\infty$, $b = +\infty$ are admissible) with the negative derivative $\dot{y}(t) = \frac{dy}{dt}$. On the complex manifold

$$E = \Big\{(\zeta, z) \in \mathbb{C} \times M \mid a < \frac{1}{2}(\zeta + \overline{\zeta}) - 2\pi F_M(z, \overline{z}) < b\Big\}, \tag{1.1}$$