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ON MULTIDIMENSIONAL METHODS OF INTERPOLATION

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A series of works are known where the authors suggest real methods, analogous to ordinary ones, but with functors acting not over a pair of spaces, but over a set of n or 2^n spaces. We shall refer methods of that sort as multidimensional. Multidimensional analogs of the real K-method were considered in [1]–[4]. Note that the appropriateness of introduction of multidimensional methods was not yet demonstrated rather convincingly, because these methods were realized only in very rare cases. In the present article we consider a simplest multidimensional method. Its functors $\mathcal{F}_{\Theta,Q}$ are composed by ordinary functors of the K-method. In the classes of spaces with two parameters this method has a convenient realization by interpolating first with respect to one parameter and then with respect to other one. For example, in interpolation in the class of Besov spaces we shall first interpolate with respect to the parameter p and then s. In doing so, at the first stage we get spaces which do not belong even to extended class of Besov spaces $B^s_{p,q,(r)}$. However, on the next stage, we again return to the class of Besov spaces. This class turns to be stable with respect to the functors $\mathcal{F}_{\Theta,Q}$, though under one-dimensional interpolation with the change of the parameter p, we, in general, leave the class of Besov spaces.

By means of multidimensional functors the interpolation theorem was obtained for the spaces B^s_p. This theorem uses weak conditions of the form

$$T : B^s_{p,1,(1)} \to B^{\tilde{s}}_{\tilde{p},\infty,(\infty)}. \tag{1}$$

Let us note that one-dimensional theorem for spaces B^s_p with weak conditions of the form (1), generally speaking, fails to be valid. An example is known with an operator satisfying weak conditions for any (p,s), $(\widetilde{p},\widetilde{s})$ with which the conditions hold $-\infty < s < +\infty$, $1 < p < \infty$, $s = \widetilde{s} = 1/p$, $p = \widetilde{p}$, but $T \notin \mathcal{L}(B^{1/r}_r, B^{1/r}_r)$ for none $r \in (1,\infty)$.

Note that, finally, in [5] an assertion can be found as if a coincidence of the methods $\mathcal{F}_{\Theta,Q}$ and that by Fernandez takes place. However, some authors (see [6], [7], [8], [4]) gave examples which show that this is not true.

1. Basic definitions and notation

Let four Banach spaces $A_{0,0}$, $A_{1,0}$, $A_{0,1}$, and $A_{1,1}$ be given embedded into one Hausdorff TVS (topological vector space) $\mathcal{A}$. On this set of four spaces $A_{i,k}$ we define the multidimensional functor

$$\mathcal{F}_{\Theta,Q}((A_{i,k})_{k=0,1})_{i=0,1} := ((A_{0,0}, A_{0,1})_{\theta_1,q_1}, (A_{1,0}, A_{1,1})_{\theta_1,q_1})_{\theta_2,q_2};$$

$\Theta = (\theta_1,\theta_2)$, $Q = (q_1,q_2)$, $0 < \theta_1,\theta_2 < 1$, $1 < q_1,q_2 < \infty$.

By a quasinormed (Banach) lattice we shall call a quasinormed (Banach) space of functions E, whose norm possesses the monotonicity property:

$$|f(x)| \le |g(x)| \ \ \forall x \Rightarrow \|f \mid E\| \le \|g \mid E\|.$$