**Fresnel phase zone plate**

Experiment Objective: explore the principle of the Fresnel phase zone plate and determine its spectral characteristics.

To calculate the diffraction patterns Fresnel proposed an approximate method based on the concept of the so-called half-wave zones (Fresnel zones).

From the definition of the Fresnel zones for the observation point $P$, it follows that the light waves emitted by the even Fresnel zones come to a point $P$ in one phase, and the odd - in the opposite. Thus, the wavelength of the odd and even zones are mutually weakened each other. If in any way to remove all even or all odd zone it will be possible to obtain a substantial increase in the total electric field strength and hence the light intensity at the point $P$. This idea is the basis of a simple optical device called a Fresnel zone plate.

![Fresnel zone plates. Left - the odd open area on the right - even.](image)

Figure 1 shows a vector diagram of field strength amplitude summation at point $P$ created by odd Fresnel zones.

It can be seen that the zone plate significantly increases the amplitude (and intensity) of the light field at the point $P$. Since the zone plate could not increase the power of the incident light wave, then it may be concluded that it redistributes energy in space, focusing it in a certain areas. In other words, the zone plate focusing light similarly to the lens.

![Fig.1. Vector diagram for a Fresnel zone plate. The vectors $E_1, E_3, E_5, \ldots$ represent the contributions to the amplitude of the total field of open odd Fresnel zones. The amplitude of the total field $E_P$ greatly exceeds the amplitude of the field $E_{\infty}$, which would be observed at the point $P$ in the absence of the zone plate.](image)
Following equation relates the distance \( a \) (Fig.2) from a point source to the zone plate and from the zone plate to the point of observation \( P \) with the radii of the Fresnel zones \( r_m \):

\[
r_m = \sqrt{\frac{ab}{a+b} m\lambda},
\]

where \( m \) - zone number, \( \lambda \) – wavelength.

Equation (1) can be rewritten as:

\[
\frac{1}{a} + \frac{1}{b} = \frac{m\lambda}{r_m^2} = \frac{1}{f},
\]

where the value \( f = \frac{r_m^2}{m\lambda} \) is the focal length of the zone plate.

A characteristic feature of the zone plate as compared with the lens is that its focal length depends on the wavelength. This phenomenon is called chromatism. The longer the wavelength, the smaller the focal distance.

Focusing effect of the zone plate will be observed in the case when in each open annular region to 3, 5, ..., etc Fresnel zones will be laid. Consequently, unlike the lens for each wavelength band plate has several foci. This situation is similar to the presence of maxima of different orders in the diffraction gratings.

Focusing effect of the zone plate will occur even when the light rays fall on the plate at a slight angle. Therefore, a picture of extended sources or subjects could be observed with the help of the zone plate.

When light passes through an amplitude zone plate half the energy of the incident light wave is lost due to absorption in the opaque areas of the plate.

To avoid these losses and increase the efficiency of light energy collection at the point \( P \) the phase zone plate is used. It modifies the phase of the oscillation of the even zones to \( \pi \). The amplitude of the resulting oscillation compared with the amplitude of the zone plate at the point \( P \) increases twice, and the intensity, respectively, quadruples (Fig.3).
Fig. 3. Profile of a two-level phase zone plate.

To change the phase of the wave at $\pi$, change the optical path for $\lambda/2$. To do this it is needed to decrease or increase the thickness of the glass plate at the positions corresponding to even Fresnel zones to the value of $h$:

$$h = \frac{\lambda}{2(n-1)},$$

(3)

The profile of such a plate is shown in Fig. 3. For visible light, the value $h$ is approximately equal to 0.5 micron.

A two-level phase zone plate is able to concentrate at the point $P$ up to 40% of light energy incident on it. Further increase of the oscillation amplitude at the point $P$ is possible by using a three-layer phase zone plate (Fig. 4).

Fig. 4. Profile of a three-layer phase zone plate (a). The right figure (b) shows a vector diagram illustrating how to determine the total amplitude of the oscillations of the first two zones.

To create such a phase zone plate, the surface of two adjacent zones, such as 1 and 2 is divided into three annular parts of equal area (obviously they will vary in width). The contribution of each part of the vector diagram represented by vectors.
OA, AB and BC, and their sum is equal to zero (Fig. 4b). To obtain the maximal modulus of the sum it is necessary to turn the vector AB through $2\pi/3$, and the vector BC through $4\pi/3$. This can be achieved by decreasing the wave phase of this areas. To do this, it is necessary to make a second annular area deeper to the value $h_{\text{max}}/3$, and third area to $2h_{\text{max}}/3$. Profile of a such phase zone plate is shown in Fig.4a. The oscillation amplitude at the point P, created by a three phase zone plate, is $\frac{3\sqrt{3}}{4}$ times greater than the amplitude produced by a two-level phase zone plate, and intensity, respectively, $27/16$ times (see Fig.3 and Fig.4).

Three-level phase zone plate is used in this paper. It highlights the nonmonochromatic light of a neon gas discharge lamps. Neon radiation in the visible region (Fig.5) is rich in lines in the red and orange regions of the spectrum (This explains the characteristic reddish glow).

![Fig.5. The emission spectrum of neon in the visible region.](image)

It was noted above that the phase zone plate has strong chromatism: for different wavelengths it has different focal lengths (see formula 2). If the light of a neon lamp direct to the phase zone plate then after the light passage of it waves of different wavelengths will gather at different points $P_i$ (Fig.6).

![Fig.6. Chromaticism of the zone plate.](image)

If a prism disperses the nonmonochromatic light into a spectrum in the direction perpendicular to the direction of propagation of the wave, the phase zone plate disperses the light into a spectrum along the direction of propagation of the wave.
Tasks:
- visually observe the focusing of the light wave for Fresnel phase zone plate;
- to determine the focal length for different wavelengths and to determine the radius of the Fresnel zones for Fresnel phase zone plate.

Optical elements and apparatus:

✓ neon lamp (1) with power supply (2);
✓ incandescent electric lamp (3) with power supply (4);
✓ phase zone plate in the holder;
✓ fiber-optic waveguide support (6);
✓ optical bench (7) with riders (8);
✓ Red Tide USB-650 spectrometer (9);
✓ an optical fiber (10) connected to a spectrometer;
✓ PC with “Spectra Suit” software.

Fig. 7 Experimental setup.

The operation principle of the Red Tide USB-650 spectrometer.

Below is a diagram of how light moves through the optical bench of an Red Tide Spectrometer.

Light enters through the optical fiber on the input slit 1 (Fig. 8). It is a narrow slit made in the opaque plate, and has a fixed width of 25 microns.
Collimating spherical mirror (2) converts divergent light beam into a parallel and directs it to a reflective diffraction grating (3). Decomposed light is focused on the detector surface (Silicon Line - CCD detector) (6) by a focusing mirror (4) and a converging lens (5). This detector includes 650 photosensitive elements (pixels), the size of which is 14 x 200 micrometers. One count of the element corresponds to the detection of 75 photons. All elements are aligned along the same straight line so that each pixel corresponds the photosensitive wavelength of a particular light. After analog to digital conversion of electric signals from pixels the digital spectrum is transmitted to the SpectraSuite program.

Setting the spectrometer and software shell.

Run the SpectraSuite software by doubleclicking on the SpectraSuite icon on the desktop. Connect the spectrometer to the computer via the USB port. Close the auxiliary windows 1 and 2 in the operating window of the program (Fig.9). The detected spectra will be output in the window 3.
Fig. 9. The operating window of the “Spectra Suit” program.

The spectra operation buttons are on the panel shown in Figure 10:

Fig. 10. Controls and Indicators.

- Zoom to maximum (350 to 1000 nm).
- Scale graph to fill window.
- Scale graph height to fill window. Useful for magnifying small y-axis detail.
- Set axes ranges manually. Useful if you just want to study a limited waveband.
- and – Zoom in and Zoom out (you can also use the mouse scroll wheel).
- – Zoom to a selected region. Useful for looking at fine detail. By clicking the left mouse button, select a rectangular area on the screen that you want to enlarge.
- – Graph panning.
- – Store Dark Spectrum.
– Store Reference Spectrum.

– Scope Mode.

– Scope Minus Dark Mode.

Depending on the choice of mode it is possible to record the optical density spectra $A$, transmittance $T = I/I_0$, the reflection coefficient $R$ and the relative luminance $I$.

The spectra input / output buttons:

– Save Spectra. Click to save data in either a Grams SPC format, JCAMP format, binary format (which only SpectraSuite can read) or tab-delimited format (can be opened in an Excel spreadsheet).

– Opens the SpectraSuite Printing dialog box.

– Copy spectral data to clipboard.

– Overlay spectral data. Overlays a previously saved spectrum onto the current graph.

– Delete overlay spectra. Deletes any spectra that have been overlaid on the current graph.

Upon spectrum registration it is important that the recorded signal does not exceed 4000 counts (one count corresponds to the 75 photons ingress on the light-sensitive matrix). So it is necessary to choose the right signal acquisition time, which is set using the "Time" button at the top left of the screen (Figure 8). The range of this parameter is 3 ms to 65 s. The longer the accumulation signal, the greater the signal. Recommended to select this option so that the signal does not exceed 3,500 counts.

Use the "averaging" button to display the average spectrum. The number of the spectra that you want to average given in the appropriate box (Figure 11).
Exercise 1. Visual observation of the focusing of the light wave by phase zone plate.

1. Mount the incandescent electric lamp (3) on the right end of the optical bench (7) (Fig.7). If the matte plate mounted on the lamp case, carefully remove it.
2. Phase zone plate (5) in the holder set at a distance of about 50 cm from the lamp.
3. Connect the incandescent lamp to the 6V power supply connectors. Connect the power supply to the network. To turn off the lamp enough to disconnect one lamp connector from power supply.
4. Take a sheet of white paper and use it as the screen to observe the light-emitting resistor image created by phase zone plate. Move the sheet along the optical axis of the system and observe how the color of the image changes. Please note that the image of the filament in the red zone is formed closer to the plate than the image of the threads in green. Give an explanation for this.
5. Move the paper closer to the zone plate. Approximately 15 cm from it thumbnail filament reappears. Give an explanation for this.

Exercise 2. Determination of the radii of the Fresnel zones for the phase zone plate.

1. Remove the lamp and place the neon lamp (1) on the right end of the optical bench (7) (Fig.7). The right edge of the lamp rider should be placed at around 100 cm.
2. Mount rider with phase zone plate at around 47 cm (to the right). Measure the distance between the source and a zone plate (a in formula 2).
3. Place the fiber-optic waveguide support (6) with optical fiber at around 7 cm.
4. Install centers of all optical elements at the same height (e.g., at a height of 20 cm from the table surface).
5. Connect the power supply to the neon lamp and turn it on.
6. Set the center of the phase zone plate on a single axis connecting the inlet tube and optical fiber (a small hole in the optical fiber holder). To do this, loosen the screw securing the zone plate. Observing the luminous gas image on the holder surface move the phase zone plate in a direction perpendicular to the optical axis and align the image with the inlet optical fiber. Ensure that the phase zone plate is perpendicular to the plane of the optical axis.
7. Turn on the computer and run the Spectra Suit program. In the Spectra Suit program window set the integration time 50 ms and the number of averaged spectra – 6.
8. Press to select a region of the spectrum from about 550 nm to 800 nm. Click to scale graph height to fill window.

9. Move the mouse cursor to a maximum of 585 nm line and click the right mouse button. You will see the cursor line. Its position on the wavelengths is shown in the lower box, the signal intensity at this wavelength displays to the right of the window. By varying the integration time you should ensure that the signal does not exceed more than 3800 counts. Watch closely for this in future measuring. If the signal is very unsteady during the acquisition, increase the number of averaged spectra.

10. Move slowly the fiber-optic waveguide support along the optical bench and find a position in which the intensity of the line to take the maximum value. This means that for a given wavelength receiver (inlet optical fibers) is at the point P. Measure the distance between the inlet of the optical fiber and a zone plate (b in formula 2).

11. Repeat steps 9 and 10 for the other 8-10 spectral lines and fill the table 1:

<table>
<thead>
<tr>
<th>(\lambda/\text{nm})</th>
<th>(10^7/\lambda)</th>
<th>(b/\text{cm})</th>
<th>(ab/(a+b)/\text{cm})</th>
</tr>
</thead>
<tbody>
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<td></td>
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12. Plot the relationship \(\frac{ab}{a+b}\) from \(10^7/\lambda\) and determine the \(\frac{r_m^2}{m}\) value. Give the values of the radii of the first areas: \(r_1, r_2, r_3\).

**Exercise 3. Registration the emission spectrum of neon at different positions of the phase zone plate.**

1. Again move the mouse cursor to a maximum of 585 nm line. Select the spectrum registration time so that signal does not exceed 3000 counts.
2. Slowly move the fiber-optic waveguide support along the optical bench and find a position in which the intensity of this line will take the maximum value. Click to save spectrum (Grams SPC format, JCAMP format, binary format (which only SpectraSuite can read) or tab-delimited format (can be opened in an Excel spreadsheet).
3. Repeat steps 1 and 2 for the 724 nm line.
4. Plot the two spectra on the same graph. Explain why there is an alteration of these lines.