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Nonholonomic Torses of the First Kind

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Abstract—In the three-dimensional Euclidean space, we study two-dimensional nonholonomic distributions with zero total curvature of the first kind, called nonholonomic torses of the first kind. The two cases are considered: 1) one of the principal curvatures of the first kind differs from zero (the general case), 2) both of the principal curvatures of the first kind equal zero (a nonholonomic plane). The result obtained in the second case is of the general form. In the study we use the canonical moving frame and apply Cartan's exterior forms method described by by S. P. Finikov in the book *Cartan's Exterior Forms Method in Differential Geometry* (GITTL, Moscow–Leningrad, 1948).

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INTRODUCTION

A two-dimensional distribution on E_3 (or in a domain $G \subset \mathbf{E}_3$) is a smooth mapping Δ assigning to each $M \in E_3$ a two-dimensional plane π passing through M ([2], P. 683). To a distribution Δ there corresponds a Pfaff equation. A distribution is called holonomic if the corresponding Pfaff equation is completely integrable. In this case, the space E_3 (or a domain G of E_3) fibers into a one-parameter family of surfaces. If the Pfaff equation is not completely integrable, the distribution is called nonholonomic ([3], P. 13), and its integral curves are called curves of the distribution. All curves of the distribution passing through a point M are tangent to the plane π at M. The pair (M, π) is called the plane element at M, and π is called the plane of the distribution at M. The set of all plane elements (M, π) (the "graph" of the distribution) is a three-dimensional manifold. This fact allows ones to use Cartan's exterior forms method. The straight line that passes through M and is orthogonal to π is called the nornal of the distribution at M. The set of all normals is a line complex.

An important field of applications of nonholonomic geometry is the dynamics of mechanical systems with nonholonomic constraints. The theory of distributions is closely connected with the geometry of unit vector fields and can be applied in the situations where such vector fields appear. For this reason, it is of importance to find solutions to various problems of nonholonomic geometry.

PRELIMINARIES

To each element (M, π), let us assign an orthonormal frame $(M, \overline{e_i})$ $(i = 1, 2, 3)$. Let $\overline{r}$ be the position vector of M and $\overline{e_3}$ the unit normal vector of the plane of the distribution at M. The derivational formulas of the frame are as follows:

$$\begin{aligned} d\overline{r} &= \omega^i \overline{e_i}, \\ d\overline{e_i} &= \omega_i^j \overline{e_j}, \end{aligned} \tag{0.1}$$

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