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MAXIMUM PRINCIPLE IN NONSMOOTH PROBLEMS OF OPTIMAL IMPULSE CONTROL WITH MULTI-POINT PHASE CONSTRAINTS

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1. Introduction

In this article necessary conditions for the first order optimality in the form of the maximum principle (MP) are obtained in the optimal control problem with trajectories of bounded variation in presence of general multi-point phase constraints. The problem of optimal impulse control is considered without assumptions on smoothness of the input data — a measurable dependence on the time and the Lipschitz property with respect to phase coordinates are assumed. In addition, we do not assume the fulfillment of well-posedness condition; this leads us to non-uniqueness of the reaction of the dynamical system to an impulse control (a vector measure). As a consequence, to an impulse control and an initial condition a funnel of generalized solutions may correspond.

Let us cite the definition of a generalized (gen.) solution, which is custom in the theory of optimal impulse processes (see [1], [2]). Consider the controllable system

$$\dot{x} = f(t,x,V,u) + G(t,x,V)v, \quad \dot{V} = \|v\|, \tag{1}$$

$$u(t) \in U, \quad v(t) \in K, \quad t \in [t_0, t_1]. \tag{2}$$

Here $x(\cdot)$, $V(\cdot)$ are absolutely continuous, $u(\cdot)$, $v(\cdot)$ measurable bounded functions, U a compact set in $R^{d(u)}$, K a convex cone in $R^{d(v)}$, $\|v\| = \sum\limits_{j=1}^{d(v)} |v_j|$, $d(z)$ stands for the dimension of the vector z. Unless otherwise stated, the terms "measurability" and "boundedness" applied to functions are related to the Lebesgue measure $\mathcal{L}$, while all the relations containing measurable functions are assumed to be fulfilled $\mathcal{L}$-almost everywhere.

Definition (see [1]). A pair of functions $x(\cdot)$, $V(\cdot)$, continuous from the right on $(t_0, t_1]$ and possessing a bounded variation, is called a generalized solution of system (1), (2) if a sequence of functions $\{x_n(\cdot), V_n(\cdot), u_n(\cdot), v_n(\cdot)\}$, which satisfy system (1), (2), exists such that

$$\sup_n \|v_n(\cdot)\|_{L_1} < \infty, \quad (x_n, V_n) \to (x, V) \ \text{ weakly}^* \text{ in } \ BV$$

(then $(x_n(t), V_n(t)) \to (x(t), V(t))$ at the points of continuity (x, V) and at the end-points of the segment T).

If we set $w_n(t) = \int\limits_{t_0}^{t} v_n(\tau)d\tau$, then from the Helly's theorem it follows that the sequence $\{w_n(\cdot)\}$ can be assumed to be weakly* convergent to a certain function of bounded variation $w(\cdot)$. In this situation, the generated Lebesgue–Stieltjes measure dw turns to be K-valued due to the convexity

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