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# METHODS FOR SOLVING LINEAR INTEGRAL EQUATION WITH KERNEL POSSESSING FIXED SINGULARITIES

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We consider the linear integral equation of the form

$$Ax \equiv x(t) + (Kx)(t) = y(t) \quad (0 \le t \le 1),$$
$$Kx \equiv \int_0^1 K(t,s)[u(s)]^{-1}x(s)ds, \quad u(t) \equiv t^{p_1}(1-t)^{p_2}, \tag{1}$$

where $p_i \in \mathbb{R}^+$ $(i = \overline{1,2})$, $K$ and $y$ are known continuous functions possessing certain properties of "smoothness" of pointwise nature, $x$ the desired function, and the integral is treated in the sense of Hadamard's finite part (see [1], pp. 144–150). To equations of the form (1) many boundary problems of the Mathematical Physics can be reduced, in particular, problems of solving loaded integral differential equations (see [2]) and those of the theory of equations of mixed type (see [3], [4]). Since the equations under investigation can be solved exactly only in rare particular cases, the development of effective methods for approximate solving these equations is of an especial actuality.

Let us note that equation (1) in the particular case $p_1 = 0$ was investigated in [2] and [5]. In [2], under rigid constraints (like the requirement of smoothness) upon $K$ and $y$, it was proved that for (1) with $p_1 = 0$, $1 < p_2 < 2$ all the Fredholm theorems are valid. In [5], on the basis of a connection between the investigated equation and an integral equation of the third kind, for (1) the Fredholm theorems were proved for $p_1 = 0$, $p_2 \in \mathbb{R}^+$.

In this article, on the basis of [6]–[10], we investigate the resolvability of equations of the form (1). Namely, we construct the Fredholm theory in the general case $(p_1, p_2 \in \mathbb{R}^+)$; suggest and substantiate in the sense of [11] (Chap. 1) a special direct method adapted to an approximate solving equation (1); we establish that the constructed method is optimal by accuracy order on the class $F$ generated by the class $H_\omega^r$, among all projection methods for solving equations of the form (1).

**1.** *On the basis space.* Let $C \equiv C(I)$ be the space of continuous on $I \equiv [0,1]$ functions with the ordinary max-norm and $p_1 \in \mathbb{R}^+$. Following [12], we write $g \in C_0^{\{p_1\}}(I) \equiv C\{p_1;0\}$ if the right Taylor derivatives $g^{\{i\}}(0)$ $(i = \overline{1,[p_1]})$ exist at the point $t = 0$, and in the case where $p_1 \ne [p_1]$ ($[\,\cdot\,]$ is the integer part), the finite limit exists:

$$\lim_{t\to 0+}\left\{\left[g(t) - \sum_{i=0}^{[p_1]} g^{\{i\}}(0)t^i/i!\right]t^{-p_1}\right\}$$

(we assume $C\{0;0\} \equiv C$).

In a similar way we define the lineal $C\{p_2;1\} \equiv C_1^{\{p_2\}}(I) \subset C$. Namely, we write $g \in C\{p_2;1\}$ if the left Taylor derivatives $g^{\{i\}}(1)$ $(i = \overline{1,[p_2]})$ exist at $t = 1$, and for $p_2 \ne [p_2]$ the following limit