Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 48, No. 1, pp. 33–44, 2004 *UDC 519.68*

ALGORITHMS IN THE PENALTY METHOD WITH APPROXIMATION OF THE FEASIBLE SET

Ya.I. Zabotin and I.A. Foukin

For solving the mathematical programming problem with the given precision, in [1] we proposed to minimize the objective function on a set immersed to the given one. The immersed set for problems with inequalities was constructed by shifting the bound of the feasible set, for which any iteration point, lying in the feasible set, was an approximate solution of the initial problem with the given precision. In contrast to the well-known method of modified Lagrange functions [2], we obtain an approximate solution only at the expense of increase of the penalty parameter up to the predetermined value and the immersed set remains fixed till the end of optimization process.

In this paper, the approximation of the feasible set is a Lebesgue set of the penalty function. It is constructed so that any minimum point of an auxiliary function which belongs to the difference of the feasible set and its approximation is a solution of the initial problem with the given precision. Using this principle, we propose two algorithms with different type of approximation of the feasible set. Similarly to [1], we estimate the penalty coefficient which necessarily provides the hit of some iteration point into the feasible set. In addition, we weaken the conditions imposed on the objective function and constraints. We also propose algorithms, using the incomplete minimization of the auxiliary function.

1. Problem definition. Condition of the ρ-approximability

Let functions $f(x)$, $f_i(x)$ for $i \in I = \{1, 2, \dots, m\}$ be defined, continuous, and convex in a certain n-dimensional Euclidean space R_n. For an arbitrary real λ we define the set $D(\lambda) = \{x : x \in R_n, g(x) + \lambda \le 0\}$, where $g(x) = \max\{f_i(x),\ i \in I\}$. We suppose that the function $f(x)$ attains its minimum on $D(0)$. Denote

$$f^* = \min\{f(x),\ x \in D(0)\}. \tag{1}$$

One needs to find a point $x' \in X^*_\varepsilon = \{x \in D(0) : f(x) - f^* \le \varepsilon\}$ for a given real $\varepsilon > 0$. We call x' an ε-solution of problem (1). From now on, we suppose that the set $D(0)$ satisfies the Slater condition, i. e., $\{x : x \in R,\ g(x) < 0\} \neq \emptyset$.

For the convenience of the results statement, we formulate the definition of (ρ, β, λ)-approximation of functions on the basis of the well-known ([3], p. 245) concept of the ρ-regularity of constraints in the mathematical programming problem. In this definition and the following lemmas we suppose that the function $\varphi(x)$ is defined in R_n, and the real λ is such that

$$M(\lambda) = \{x : x \in R_n,\ \varphi(x) \le \lambda\} \neq \emptyset,$$

G is the given set in R_n, $M(\lambda) \cap G \neq \emptyset$ and, as usual, $\rho(x, M(\lambda)) = \inf_{y \in M(\lambda)} \|x - y\|$.