Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 48, No. 1, pp.28–32, 2004 *UDC 517.988:519.677*

SOLUTION METHOD FOR SEMICOERCIVE VARIATIONAL INEQUALITIES BASED ON THE METHOD OF ITERATIVE PROXIMAL REGULARIZATION

E.M. Vikhtenko and R.V. Namm

In this paper, we investigate variational inequalities in Mechanics, in which the condition of strict coercivity (strong convexity) of the corresponding minimized functional holds only on the subspaces of finite co-dimension of the initial Hilbert space. The series of important, from the practical point of view, problems of Mechanics of Continua can be reduced to these semi-coercive variational inequalities; for example, the contact problem in the theory of elasticity, when one of the contacting objects is not fixed ([1], p. 112; [2], p. 274).

Semicoercive variational inequalities in Mechanics accept one general abstract scheme which allows us to construct and validate solution methods for such inequalities based on the iterative proximal regularization.

So, let the two Hilbert spaces V, H be given, $H \subset V$, H_1 be a finite-dimensional subspace of H, $Q_1 : H \to H_1$ be an orthoprojection, $Q_2 = I - Q_1$, where I is an identity operator.

Now we suppose that the convex finite-valued functional $\gamma : H \to R$ is given such that

$$\gamma(\lambda v_1 + (1-\lambda)v_2) \le \lambda\gamma(v_1) + (1-\lambda)\gamma(v_2) - \delta\lambda(1-\lambda)\|Q_2 v_1 - Q_2 v_2\|_H^2, \quad 0 \le \lambda \le 1. \qquad (1)$$

It is not difficult to see that property (1) means that the functional γ has the property of strong convexity on the subspace $H_2 \equiv Q_2 H$ with the constant $\delta > 0$.

Suppose that the value $|||u|||_H^2 \equiv \|Q_2 u\|_H^2 + \|u\|_V^2$ is equivalent to $\|u\|_H^2$.

We consider the extremal problem

$$\gamma(u) \ \to \ \min, \quad u \in G, \qquad (2)$$

where G is a convex closed subset of H. For solving problem (2), we consider the method of iterative proximal regularization:

a) choose an arbitrary element $u^0 \in H$;

b) denote $\overline{u}^{k+1} = \arg\min\limits_{u\in G}\{\gamma(u) + \|u - u^k\|_V^2\}$, calculate u^{k+1} by the criteria

$$\|u^{k+1} - \overline{u}^{k+1}\|_H \le \varepsilon_{k+1},$$

where $\{\varepsilon_k\}$ is the given sequence of positive numbers.

Below we will show that the regularizing addition $\|u - u^k\|_V^2$ to the functional $\gamma(u)$ generates the strongly convex on H functional $\psi_{k+1} = \gamma(u) + \|u - u^k\|_V^2$. This fact provides the unique existence of the element $\overline{u}^{k+1}$ for all $k = 0, 1, 2, \dots$ and allows us to use efficient numerical methods for finding approximate solutions u^{k+1}.

Note that an analogous scheme of the iterative prox-regularization was given in [3], for the functionals, presentable in the form of $\gamma(u) = \gamma_1(Q_1 u) + \gamma_2(Q_2 u)$, where γ_1 is a convex functional on $H_1 \equiv Q_1 H$, and γ_2 is a strongly convex functional on $H_2 \equiv Q_2 H$. These functionals belong to