*Russian Mathematics (Iz. VUZ)* *Izvestiya VUZ. Matematika*
*Vol. 45, No. 9, pp. 62–70, 2001* *UDC 517.958*

# INVESTIGATION OF SOLVABILITY OF VARIATIONAL PROBLEMS IN THE NONLINEAR THEORY OF THIN SHELLS

S.N. Timergaliyev

The objective of this article is to study the solvability of variational problems of both geometrically and physically nonlinear theory of thin non-sloping shells of zero Gaussian curvature with rigidly fixed edge. The characteristic property of this article is the fact that the solvability of problems is proved in a functional space which differs from spaces of displacements and forces. Solvability conditions of the problem are obtained without assumptions on smallness of components of the outer load.

Similar problems for sloping shells were considered in [1]. The physically nonlinear problems for non-sloping shells whose middle surface is formed by rotation, a convex developable surface, were studied in [2], [3].

## 1. Statement of problem

As is known (see, e. g., [4], pp. 94–95), the boundary value problem of the theory of shells can be formulated as the problem of determination of the minimum of the functional $\Phi = U - A$ of complete energy of the shell–outer forces system, where

$$U = \iiint\limits_V \Pi D^* d\alpha^1 d\alpha^2 d\alpha^3 \tag{1}$$

is the potential energy accumulated in the whole volume $V$ of the shell,

$$A = \iint\limits_\Omega [\mathcal{R}^j w_j\big|_{j=\overline{1,3}} - \mathcal{L}^i w_{3\alpha^i}\big|_{i=1,2}] D d\alpha^1 d\alpha^2 \tag{2}$$

is the work of outer "dead" forces (here and in what follows summation is carried out over repeating indices). Here $\Pi$ is the density of potential energy of deformation, defined by the formula (see [4], p. 469)

$$\Pi = \int\limits_{(0,0,0)}^{(\varepsilon_{11},\varepsilon_{12},\varepsilon_{22})} \sigma^{11} d\varepsilon_{11} + 2\sigma^{12} d\varepsilon_{12} + \sigma^{22} d\varepsilon_{22}, \tag{3}$$

where the integral is taken along the path of loading and does not depend on the integration path (ibid., p.470); $\sigma^{\lambda\mu} = \sigma^{\lambda\mu}(\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{22})$ are components of the stress tensor; $\mathcal{R}^j$, $\mathcal{L}^i$ are components

Supported by the Russian Foundation for Basic Research, project no. 99-01-00410, and Academy of Sciences of Republic of Tatarstan.