Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 46, No. 3, pp.38–46, 2002 *UDC 512*

OPERADS IN THE CATEGORY OF CONVEXORS. I

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In [1]–[3], a class of universal algebras called convexors (or baricentric algebras) was studied. Some examples of these algebras are well-known: the convex subsets in Euclidean spaces (in particular, the sets of stochastic matrices) and upper semilattices. We also can refer, for example, to the applications in the theory of probabilistic automata. In this article we study connections existing between convexors and operads. Seemingly, as a subject of study, linear operads first appeared (with another name) in [4]. The term "operad" was introduced in [5] (see also [6]). As for the present state of the operad theory, we refer the reader to [7]–[10]. In [11], [12], it was demonstrated that the class of varieties of linear (multioperator) algebras over linear operads coincides exactly with the class of varieties of multioperator linear algebras determined by multilinear identities. Note that (abstract) clones represent objects similar to operads. Relations between these concepts were described in [13].

In the first part of this article we give a new definition of an operad, more general than the generally accepted one. We study in detail the operad whose components are standard simplexes (with dimension shifted by one). Theorem 1 states that the variety of convexors is rationally equivalent to the variety of algebras over that generalized operad. Farther we consider some details of the structure of the category of convexors. Note that the category Conv, studied in this article differs significantly from the category STOCH which was the main subject of study in [1], [2].

In the second part of the article we will introduce a convexor analog of a linear operad: a conoperad, i. e., an operad all of whose components are convexors and whose operations of composition are multilinear in the sense of the theory of convexors. The main result of the article is the characterization of varieties of conalgebras over conoperads: these are exactly those varieties which are determined by convexor analogs of multilinear identities. Thus, the situation is identical to that in the linear case. The results of the article were announced in [14].

An operad can be defined as a generalization of a category in which "arrows" have not one "origin" (object) but several ones. The exact definition is as follows. An operad $\mathfrak{R}$ is the following set of data. First, a class of "objects" $S = \mathrm{Ob}\,\mathfrak{R}$ is given. Second, for each nonempty word $\overline{x} = x_1 \dots x_n$ in the alphabet S and each $y \in S$, a set of "arrows" $\mathfrak{R}(\overline{x}, y)$ (possibly, empty) is given. Finally, for nonempty sets of "arrows", an operation of composition

$$\mathfrak{R}(y_1 \dots y_m, z) \times \mathfrak{R}(\overline{x}_1, y_1) \times \dots \times \mathfrak{R}(\overline{x}_m, y_m) \longrightarrow \mathfrak{R}(\overline{x}_1 \dots \overline{x}_m, z) \qquad (1)$$

is defined, which is denoted by $(\omega, \omega_1, \dots, \omega_m, \omega) \mapsto \omega\omega_1 \dots \omega_m\omega = \omega(\omega_1 \dots \omega_m)$. Here $\overline{x}_i = x_{i\,1}x_{i\,2} \dots x_{i\,n_i}$, $\omega_i \in \mathfrak{R}(\overline{x}_i, y_i)$, $1 \le i \le m$, $\omega \in \mathfrak{R}(y_1 \dots y_m, z)$. This can be represented in the form

Supported by the Russian Foundation for Basic Research (grant no. 99-01-00469).