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CONVOLUTIONS INDUCED BY AUTOMORPHISMS OF LINEAR STRUCTURE OF ALGEBRA

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In what follows we consider a scheme in which an integral transform subject to inversion is interpreted as a multiplier in a certain ("convolutional") algebra. In this situation the proper problem of inversion turns to be equivalent to the description of the group of units of this algebra. To "convolutional" algebras rather extended literature is devoted, therefore the author restricts himself to inclusion into the references only the works [1]–[7] related to this theme, which were available for him.

1. *The algebra $A_{\sigma(*)}$.* Consider a certain commutative associative algebra with unit over the field k. We denote by $*$ the multiplication in this algebra, while the algebra is denoted by A_*. Let σ be an automorphism of the appropriate linear space A of the algebra A_*. We define in A a new multiplication $\sigma(*)$ (*σ-convolution*) via the rule

$$x\sigma(*)y = \sigma^{-1}(\sigma(x) * \sigma(y)), \quad x, y \in A. \tag{1}$$

The arising algebraical system is denoted by $A_{\sigma(*)}$. One can easily see that $A_{\sigma(*)}$ is an associative algebra with unit over the field k. Let U_* and $U_{\sigma(*)}$ be, respectively, the group of units of algebras A_* and $A_{\sigma(*)}$. If $x \in A_*$ and $x \in A_{\sigma(*)}$, then $x^{(-1)}$ and $x^{[-1]}$ stand for the inverse element for x in the respective algebra.

If τ is a certain other automorphism of the linear space A and $A_{\tau(*)}$ is the corresponding algebra, then, obviously, both the algebras are *isomorphic*.

Let us fix an element $a \in A_{\sigma(*)}$ and define the mapping $R_\sigma(a) : A_{\sigma(*)} \to A_{\sigma(*)}$, by assuming $R_\sigma(a)(x) = x\sigma(*)(a)$. Clearly, $R_\sigma(a)$ is a linear operator. In what follows it will be called a *multiplier* of the algebra $A_{\sigma(*)}$. It is invertible if and only if $a \in U_{\sigma(*)}$; in addition, $R_\sigma^{-1}(a) = R_\sigma(a^{[-1]})$, or in terms of values $R_\sigma(a^{[-1]})(x) = x\sigma(*)\sigma^{-1}(\sigma(a)^{(-1)})$, and the inversion formula for the transformation realized by $R_\sigma(a)$ has the form

$$x = R_\sigma(a)(x)\sigma(*)\sigma^{-1}(\sigma(a)^{(-1)}).$$

2. *Examples.* For the analysis, functional algebras over a certain field, in particular, over the field $\mathbb{C}$, are of interest. Let us give some examples of that kind.

2.1. Example 1. Let X be an arbitrary set, $A. = A.(X)$ a certain algebra of $\mathbb{C}$-valued functions on X, σ a fixed automorphism of the linear space A of this algebra. In this case, the operator $R_\sigma(a) : A_{\sigma(\cdot)} \to A_{\sigma(\cdot)}$ is invertible if and only if the function $\sigma(a)$ has no zeros. In this case, the inversion formula takes the form

$$f = \widehat{f}_a\sigma(\cdot)\sigma^{-1}(1/\sigma(a)),$$

where we denoted by $\widehat{f}_a$ the image $R_\sigma(a)(f)$. To turn this example more specific, we set $X = k^n$, where k is either the field $\mathbb{R}$, or $\mathbb{C}$, or a locally compact field, or a finite filed; $A.$ the algebra of basic functions on k^n (in the case of a finite field, it is the algebra of all $\mathbb{C}$-valued functions on k^n). In