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OPTIMAL BY ORDER CONVERGENCE RATES OF A PIECEWISE UNIFORM GRID FOR SINGULARLY PERTURBED EQUATIONS OF CONVECTION–DIFFUSION TYPE

G.I. Shishkin

1. Introduction

To the present time special numerical methods are sufficiently developed for the singularly perturbed boundary value problems. In contrast to the methods developed for solving regular boundary value problems (see [1], [2]), these methods allow us to find grid solutions converging uniformly with respect to a perturbing parameter ε (or ε-uniformly). In the construction of the difference schemes which converge ε-uniformly the fitting methods and methods of condensing grids are traditionally used (see, e. g., [3]–[10] and bibliography therein).

The advantage of the fitting methods (see [4], [5]) is in the possibility to use the simplest *uniform* grids. However, these schemes possess rather restricted domain of applicability. As was shown in [6], for singularly perturbed problems with parabolic boundary layer no fitting schemes exist which could converge on uniform grids ε-uniformly; consequently, the application of grids condensing in the boundary layer is a necessary condition for attainability of the ε-uniform convergence.

In the method of condensing grids on Bakhvalov's grids (see [3]), the step of the mesh varies gradually. Grids of that sort in the case of problems of convection–diffusion type allow us to obtain the first order of ε-uniform rate of convergence (see [11]) — the same as in regular boundary value problems (with the use of monotone approximations with first directed difference derivatives, see [2]). The piecewise uniform (most simple) grids condensing in boundary layer were used in [6], [7], [10], [12] (see also the bibliography in [6]–[10]). Difference schemes on these grids in the case of convection–diffusion problems converge ε-uniformly with the first order (up to a logarithmic cofactor which grows as the number of grid nodes increases). Thus, the ε-uniform rate of convergence turns to be lower than the convergence rate in the case of regular boundary value problems.

As a result of this circumstance, it seems to be of interest to improve the piecewise uniform grids from [6], [7], [10], [12] in order to augment the ε-uniform order of convergence. A special interest appears in the construction of improvable grids on the class of piecewise uniform grids. At the present time, various versions of advanced grids are suggested, however, with a gradually improving step in the layer (see, e. g., [13]–[15], and also the survey in [16]); the results were obtained, basically, for ordinary differential equations in ε-weighted energetic norms (in those norms the boundary layer (finite in the norm of l_∞) in case of partial equations tends to zero as the parameter ε tends to zero). Practically, an improvement of the grids from [6], [7], [10], [12] on classes of piecewise uniform grids was not considered; we can note only [17], in which a family of optimal (with respect to the order of the convergence rate) piecewise uniform grids was investigated for problems with prevailing convection.

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