

ON UNIQUENESS THEOREMS FOR THE EXTERIOR  
INVERSE BOUNDARY VALUE PROBLEM IN SUBCLASSES  
OF UNIVALENT FUNCTIONS

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It is known (see [1]) that the resolvability of the exterior inverse boundary value problem (see [2], § 33; [3], § 3) is reduced to the search of extremal point of inner radius of a certain domain. The uniqueness of the solution of the exterior problem is ensured by the uniqueness of the extremal point, the latter is guaranteed by certain subclasses of regular functions.

In the present article the uniqueness theorem is established in a proper subclass of star-like functions and functions with a bounded rotation. This theorem is formulated in Section 4 on the base of two sets of exact estimates, which are obtained in Sections 2 and 3. In Section 1 we give preliminary information and compare known results. In Section 4 we, in addition, pose some equation problems and trace ways of their investigation.

1.

The exterior inverse boundary value problem in F.D. Gakhov's statement (see [2], § 33; [3], § 3) has a solution of the form

$$F(\zeta) = \int_a^\zeta f'(\zeta) \left( \frac{1 - \bar{\zeta}_0 \zeta}{\zeta - \zeta_0} \right)^2 d\zeta, \quad |\zeta| < 1, \quad (1)$$

besides,  $a$ ,  $|a| < 1$ , is a complex constant,  $f(\zeta)$  is continuous in the closed circle  $\bar{E}$ , the function  $\ln |F'(e^{i\theta})| = \ln |f'(e^{i\theta})| = p(\theta)$  can be expressed via initial data,  $\zeta_0$  is a pole of the function  $F(\zeta)$ . To determine the point  $\zeta_0$  one can use the equation

$$0 = \operatorname{res}_{\zeta_0} F'(\zeta) = f''(\zeta_0)(1 - |\zeta_0|^2)^2 - 2\bar{\zeta}_0 f'(\zeta_0)(1 - |\zeta_0|^2),$$

after an easy transformation and rejection of the index near  $\zeta_0$ , the latter can be written as follows

$$f''(\zeta)/f'(\zeta) = 2\bar{\zeta}/(1 - |\zeta|^2). \quad (2)$$

In the theory of inverse boundary value problems, equation (2) is called F.D. Gakhov's equation. The uniqueness of the solution of this equation means the uniqueness of the solution of the inverse boundary value problem. Some works were dedicated to determination of the uniqueness conditions in the form of restrictions upon the function  $p(\theta)$ ; their list can be found in [4] and in the survey [5]. Later on some interesting works appeared; they are listed in [6]. From this list we select the paper [7] with its elegant result, which will be soon recalled.

The conditions for uniqueness of solution of the exterior inverse boundary value problem or the uniqueness of the extremum of the inner radius of the domain  $f(E)$  ( $E$  is the unit circle) [1] can be divided into inner conditions (with respect to the behavior of the function  $f(\zeta)$ ) and outer conditions (with respect to the behavior of the function  $F(\zeta)$  with representation (1)). The exterior