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Lie Sheaves of Small Dimensions

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Abstract—We obtain a classification of 2- and 3-dimensional Lie sheaves.

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Lie sheaves were introduced in [1], where a classification of irreducible Lie sheaves has been obtained. Further the construction of Lie sheaves was used for the study of integrability of Hamiltonian systems (see [2], P. 606; [3], P. 237). In particular, the equations of motion of an n-dimensional solid body are Hamiltonian with respect to a family of compatible Poisson brackets which are constructed on the dual space by a Lie sheaf defined on a space L. Therefore, the central functions of each bracket from this family are first integrals of the initial system of equations.

On the other hand, in [4], n-tuple Lie algebras were introduced. For these algebras, in [4, 5], analogs of the Engel and Lie theorems have been proved. For $n = 2$, the construction of an n-tuple Lie algebra has been considered in [6] under the name of a bi-Hamiltonian operad. As one can see from the definitions given below, the notions of a Lie sheaf and an n-tuple Lie algebra coincide. For this reason, in the present paper, we use the term "n-tuple Lie sheaf". The adjective "n-tuple" is convenient for formulations of results.

Below we give the definitions.

If L is a vector space over a field P, then by K we denote the space of all skew-symmetric mappings from $L \times L$ to L.

Definition 1. A vector space L over a field P is called an *n-tuple Lie sheaf* if there exist n linearly independent elements s_i, $i = 1, \dots, n$, of K such that, for any $a, b, c \in L$ and any two mappings s_i and s_j,

$$J(a, b, c, s_i, s_j) + J(a, b, c, s_j, s_i) = 0, \tag{1}$$

where $J(a, b, c, s_i, s_j) = (as_ib)s_jc + (bs_ic)s_ja + (cs_ia)s_jb$. Here xs_ky denotes the image of a pair $(x, y) \in L \times L$ under a mapping s_k.

This definition, with the change of the term "n-tuple Lie sheaf" by the term "n-tuple Lie algebra" was used in [4, 5].

For mappings s_i, $i = 1, \dots, n$, from K, we define a mapping $s = \sum\limits_{i=1}^{n} \alpha_i s_i$, $\alpha_i \in P$, acting by the rule $asb = \sum\limits_{i=1}^{n} \alpha_i(as_ib)$. By S we denote the linear span of the mappings $s_1, \dots, s_n$. One can easily see that $J(a, b, c, s, \widetilde{s}) + J(a, b, c, \widetilde{s}, s) = 0$, $a, b, c \in L$, for any two elements s and $\widetilde{s}$ from S. What is more, if the characteristic of P is different from two, relations (1) are equivalent to the relations

$$(asb)sc + (bsc)sa + (csa)sb = 0, \quad a, b, c \in L, \tag{1'}$$

where s runs through the entire space S.

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