

# THE REMARKABLE EXPRESSIVITY OF FIRST-ORDER LOGIC IN PROFINITE GROUPS

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Profinite groups are the compact totally disconnected groups, or equivalently, the inverse limits of finite groups. First-order logic in the language of groups can only indirectly talk about the topological structure.

We address the question whether a profinite group  $G$  can be determined by a single first-order sentence. That is, we ask whether there a sentence  $\phi$  such that  $H \models \phi$  iff  $H$  is topologically isomorphic to  $G$ , for each profinite group  $H$ .

Let  $p \geq 3$  be a prime. We show that this property holds for the groups  $\mathrm{SL}_2(\mathbb{Z}_p)$  and  $\mathrm{PSL}_2(\mathbb{Z}_p)$  where  $\mathbb{Z}_p$  is the ring of  $p$ -adic integers. If we restrict the reference class to the inverse limits of  $p$ -groups, we obtain many further examples, e.g. all groups with a bound on the dimension of the closed subgroups (such as the abelian group  $\mathbb{Z}_p$ ).

We will discuss both algebraic and model theoretic methods to show such results. This is joint work with Dan Segal. We will also cover upcoming work with K. Tent in the model theoretic direction.

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