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Approximate Analytic Solution of Heat Conduction Problems with a Mismatch Between Initial and Boundary Conditions

E. V. Stefanyuk* **and V. A. Kudinov****

Samara State Technical University, ul. Molodogvardeiskaya 244, Samara, 443100 Russia

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Abstract—We consider a heat conduction problem for an infinite plate with a mismatch between initial and boundary conditions. Using the method of integral relations, we obtain an approximate analytic solution to this problem by determining the temperature perturbation front. The solution has a simple form of an algebraic polynomial without special functions. It allows us to determine the temperature state of the plate in the full range of the Fourier numbers ($0 \le \mathsf{F} < \infty$) and is especially effective for very small time intervals.

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It is known that solutions to boundary-value problems of heat conduction obtained by exact analytical methods are represented in the form of infinite series that converge poorly near boundary points and for small Fourier numbers. According to the results of our research, in the range $10^{-12} \le \mathsf{F} \le 10^{-7}$ of the Fourier numbers the exact analytic solution to the nonstationary heat conduction problem for an infinite plate with the first-kind boundary conditions [1] gives a good approximation, when the number of used terms of the series ranges from 1000 to 500000.

The methods of integral relations (integral heat balance methods) [2–8] allow one to avoid the mentioned difficulties. However, their widespread use is constrained due to the inaccuracy of obtained solutions. Many attempts to increase the accuracy appeared to be unsuccessful.

Below we describe a method of integral relations that allows to obtain an approximate analytic solution to the boundary-value problem, whose precision is sufficient for engineering applications. This method works in the full time range of the nonstationary process ($0 \le \mathsf{F} < \infty$) without any constrains on the smallness of the Fourier number. Let us illustrate the basic idea of the method with an example of a heat conduction problem with a linear distribution of initial temperatures through the plate thickness. We assume that the line of initial temperatures is inclined so that at $x = 0$ ($\tau = 0$) the initial temperature attains its maximal value T_0, and at $x = \delta$ it attains its minimal value T_w (the wall temperature at $x = 0$ and $x = \delta$ is the boundary condition of the first kind). In the considered case the problem allows the following mathematical statement:

$$\frac{\partial T(x,\tau)}{\partial \tau} = a\frac{\partial^2 T(x,\tau)}{\partial x^2} \quad (\tau > 0,\ \ 0 \le x \le \delta), \tag{1}$$

$$T(x,0) = T_0 - \frac{x}{\delta}(T_0 - T_\mathrm{w}), \tag{2}$$

$$T(0,\tau) = T(\delta,\tau) = T_\mathrm{w}, \tag{3}$$

where T is the temperature, x is the coordinate, τ is the time, a is the heat conduction coefficient, T_0 is the initial temperature, T_w is the wall temperature, and δ is the plate thickness.

Introduce the following dimensionless values:

$$\Theta = (T - T_\mathrm{w})/(T_0 - T_\mathrm{w}), \quad \xi = x/\delta, \quad \mathsf{F} = a\tau/\delta^2.$$

*E-mail: stef-kate@yandex.ru.
**E-mail: kud-samgtu@yandex.ru.