ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 2, pp. 11–15. © Allerton Press, Inc., 2010.
Original Russian Text © S.R. Enikeeva, 2010, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2010, No. 2, pp. 14–19.

A Direct Method for Solving Integral Equations

S. R. Enikeeva[1]

[1] *Kazan State Agrarian University, ul. K. Marksa 65, 420015 Kazan Russia*

Received October 22, 2007

Abstract—In this paper we theoretically justify the Bogolyubov–Krylov method for weakly singular integral equations.

DOI: 10.3103/S1066369X10020027

Key words and phrases: *weakly singular integral equations, spline-collocation method, Bogolyubov–Krylov method.*

1. INTRODUCTION

For integral equations with smooth periodic kernels academicians N. N. Bogolyubov and N. M. Krylov proposed an efficient direct solution method (see, for example, [1]). In recent years this method has been applied to various integral equations with discontinuous kernels (see, for example, works [2–12] and references therein). Below we theoretically justify the Bogolyubov–Krylov method for the following integral equation frequently encountered in applications:

$$Kx \equiv x(t) + \lambda \int_{-1}^{1} \frac{\ln|\tau - t|}{(1-\tau)^{\alpha}(1+\tau)^{\beta}} x(\tau)d\tau + \mu \int_{-1}^{1} \frac{h(t,\tau)}{(1-\tau)^{\alpha}(1+\tau)^{\beta}} x(\tau)d\tau = y(t), \quad -1 \leqslant t \leqslant 1, \quad (1)$$

where $y(t) \in C[-1,1]$ and $h(t,\tau) \in C[-1,1]^2$ are given functions, $x(t)$ is the desired function, parameters $\alpha, \beta \in (-1,1)$ and $\lambda, \mu \in \mathbb{R}$, $\lambda^2 + \mu^2 \neq 0$, the integrals being understood as improper ones. Our research is based on a technique proposed by B. G. Gabdulkhaev for studying spline methods for solving regular and singular integral equations (see, for example, [2–10]).

2. A ZERO-ORDER SPLINE-COLLOCATION METHOD

On the segment $[-1,1]$ we construct an arbitrary mesh of $n+1$ nodes

$$\Delta_n : -1 = t_0, t_1, \ldots, t_{n-1}, t_n = +1, \quad n \in \mathbb{N}, \quad (2)$$

with the following restriction imposed on the mesh norm:

$$\|\Delta_n\| = \max_{1 \leqslant k \leqslant n} (t_k - t_{k-1}) \to 0, \quad n \to \infty.$$

We also introduce an auxiliary mesh with the nodes

$$\bar{t}_j = \frac{t_{j-1} + t_j}{2}, \quad j = \overline{1,n}. \quad (3)$$

Denote by $\psi_k(t) = \psi_{k,n}(t)$, $k = \overline{1,n}$, the fundamental zero-degree splines on the mesh with nodes (2):

$$\psi_1(t) = \begin{cases} 1 & \text{for } t \in [-1, t_1], \\ 0 & \text{for } t \notin [-1, t_1]; \end{cases}$$