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PROBLEM ON CONJUGATION OF SOLUTIONS OF NONSTATIONARY HEAT EQUATION IN THREE-DIMENSIONAL REGIONS WITH NONSMOOTH BOUNDARIES

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Applied problems of mathematical physics, namely investigation of nonstationary temperature fields, and dynamic temperature stresses in nonhomogeneous bodies with nonsmooth boundary surfaces make necessary to solve the corresponding conjugation problems [1].

Setting of Problem. Suppose that in the three-dimensional space R^3 there is given a finite simply connected region V_1 which is bounded by a nonsmooth surface S containing nonintersecting smooth curves of singular (angular) points and conic points.

Let us construct solutions $T_j = T_j(x, y, z, t)$ of the non-stationary heat equation [1]

$$\Delta T_j - a_j^2 \frac{\partial T_j}{\partial t} = f_j(x, y, z, t) \tag{1}$$

with the initial condition

$$T_j(x, y, z, t)|_{t=0} = 0 \tag{2}$$

and with the following conjugation conditions on the surface S ([1], p. 98).

The conjugation conditions at the smooth points are

$$\begin{gathered} T_0^-(x, y, z, t) - T_1^+(x, y, z, t) = 0, \\ \lambda_0 \frac{\partial T_0^-(x, y, z, t)}{\partial n} - \lambda_1 \frac{\partial T_1^+(x, y, z, t)}{\partial n} = 0, \end{gathered} \tag{3}$$

and the conjugation conditions at the singular points $M_0(x_0, y_0, z_0)$ are

$$\begin{gathered} \lim_{M \to M_0} [T_0^-(x, y, z, t) - T_1^+(x, y, z, t)] = 0, \\ \lim_{M \to M_0} \left[\lambda_0 \frac{\partial T_0^-(x, y, z, t)}{\partial n} - \lambda_1 \frac{\partial T_1^+(x, y, z, t)}{\partial n}\right] = 0, \end{gathered} \tag{4}$$

where $j = 0$ corresponds to values defined in $V_0 = R_3 \setminus V_1$, and $j = 1$ to the values defined in V_1; n is the outer with respect to V_1 normal vector of S, the upper indices "$\pm$" denote the boundary values of functions when the point $M(x, y, z)$ approaches S from V_1 ("+"), or from V_0 ("−"), and $\lambda_j \in R$.

For a singular point M_0, the heat equation means that the following equality holds ([2], p. 187)

$$\lim_{\Delta V \to M_0} \iiint\limits_{\Delta V} \left[\Delta T_j - a_j^2 \frac{\partial T_j}{\partial t} - f_j(x, y, z, t)\right] dv = 0, \tag{5}$$

where the limit is taken as the elementary region ΔV collapses to M_0.