

Estimation of Statistical Characteristics of Attainability Sets of Controllable Systems

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Received July 23, 2012

Abstract—We study an expansion of the notion of invariance for sets with respect to controllable systems and differential inclusions. Namely, we study statistically invariant sets and statistical characteristics of attainability sets of controllable systems. We obtain a lower bound for the lower relative frequency of the absorption of the attainability set of a system by a given set and establish new sufficient conditions of the statistical invariance of the set with respect to the controllable system. We give examples of the calculation of statistical characteristics for the linear Cauchy problem and a linear controllable system with almost periodic coefficients.

DOI: 10.3103/S1066369X13110029

Keywords and phrases: *controllable systems, dynamical systems, differential inclusions, statistically invariant sets.*

INTRODUCTION

One of important problems in the theory of controllable processes consists in studying the invariance of sets of relatively different controllable systems and differential inclusions. This topic was considered by N. N. Krasovskii, A. B. Kurzhanskii, J.-P. Aubin, A. I. Subbotin, N. N. Subbotina, E. L. Tonkov, V. N. Ushakov, T. F. Filippova, Ph. Hartman and many other authors. The number of publications dedicated to studying sets which are not invariant is essentially less [1, 2]. In these papers one introduces and studies the notion of the defect of invariance for sets without the invariance property. In [3–6] one also studies sets which are not invariant in the “classical” sense and for such sets introduces a natural generalization of the notion of invariance, namely, the statistical invariance. A set M is called statistically invariant with respect to a controllable system, if the relative absorption frequency of the attainability set $D(t, X)$ of the controllable system by the set M equals one.

This paper is a continuation of [3–7] which are dedicated to the study of such characteristics of the attainability set $D(t, X)$ of the controllable system, as the relative absorption frequency, the upper and lower relative absorption frequencies of the set $D(t, X)$ defined by the set M . In this paper we obtain estimates of these characteristics for controllable systems with almost periodic coefficients. We also consider the following problem: for a given number $\lambda_0 \in (0, 1]$ it is necessary to find conditions for the controllable system and the set X guaranteeing that the relative absorption frequency of the attainability set $D(t, X)$ of the system by the set M is no less than λ_0 .

1. BASIC DEFINITIONS

In this work we study statistical characteristics of the attainability set of the controllable system

$$\dot{x} = f(t, x, u), \quad (t, x, u) \in \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m. \quad (1.1)$$

Consider the differential inclusion corresponding to system (1.1)

$$\dot{x} \in F(t, x), \quad F(t, x) = \text{co } H(t, x), \quad (1.2)$$

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