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A DIFFERENTIAL MANY-PERSON GAME WITH INTEGRAL CONSTRAINTS ON PLAYERS' CONTROLS

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1. Introduction

In [1]–[4], the fundamental results of the theory of differential games were described.

N.N. Krasovsky has given the mathematical formalization of the notion of a differential game. Using it, he, together with A.I. Subbotin [5] and other collaborators, have proven the theorem of alternative and proposed the effective methods for constructing the extremal strategies of approaches and evasions according to the so-called extremal aiming rule.

Another approach to the definition of a differential game has been proposed by L.S. Pontryagin. Particular features of this method consist in consideration of a differential game from the point of view of a certain player and the informational discrimination of the opponent.

This paper follows the formalization proposed by L.S. Pontryagin. Differential games with integral constraints on players' controls represent the particular interest. Such games have been investigated in [6]–[13]. The fundamental results in this realm have been obtained in [6] and [7]. In [8]–[12], the problems of pursuit one object by several ones are investigated.

We consider the controllable system

$$\dot{z}_{ij} = C_{ij} z_{ij} + u_i - v_j, \quad z_{ij}(t_0) = z^0_{ij}, \quad i = 1, 2, \dots, k, \quad j = 1, \dots, m, \tag{1}$$

where $z_i \in R^n$, C_{ij} is a constant $n \times n$-matrix, u_i is a control parameter of pursuit, v_j is a control parameter of evasion. We choose the parameter u_i as a function $u_i = u_i(\cdot)$ from the closed sphere $S(\rho_i)$ of the radius ρ_i centered at zero of the space $L_2[t_0; \infty)$. We choose the parameter v_j as a function $v_j = v_j(\cdot)$ from the sphere $S(\sigma_j) \subset L_2[t_0, \infty)$. We call such functions the feasible controls of pursuers and escapees. If $u_i(\cdot)$, $i \in \{1, 2, \dots, k\}$, and $v_j(\cdot)$, $j \in \{1, 2, \dots, m\}$, are the feasible controls of the i-th pursuer and the j-th escapee, respectively, then the trajectory $z_{ij}(\cdot)$ is defined as an absolutely continuous solution of the Cauchy problem

$$\dot{z}_{ij} = C_{ij} z_{ij} + u_i(t) - v_j(t), \quad z_{ij}(t_0) = z^0_{ij}.$$

The aim of the i-th pursuer is to attain the equality $z_{ij}(t) = 0$ for the least time t, and the aim of the j-th escapee is to prevent it. We consider game (1) to be complete if for any $j \in \{1, 2, \dots, m\}$ there exists $i \in \{1, 2, \dots, k\}$ such that $z_{ij}(t_{ij}) = 0$ for a certain $t_{ij} \geq t_0$.

Definition 1. A function

$$u_i(t, \xi_i, v_1, \dots, v_m), \quad u_i : [t_0; \infty) \times [0, \rho_i^2] \times R^n \times \dots \times R^n \to R^n$$