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A Two-Point Boundary-Value Problem for Gyroscopic Systems in Some Lorentzian Manifolds

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Abstract—We study the dynamics of gyroscopic systems of relativistic type with multivalued action functionals. We assume that Lorentzian configuration manifolds have the structure of the twisted product. The solvability of the two-point boundary-value problem for such systems was proved earlier only in the case of a limited Lorentzian distance from the initial point to the final one. In this work we obtain a new existence theorem. According to this theorem, the specified distance to attainable points may be arbitrarily large. The result is applied to the dynamics of a charged test particle in the external space-time of the Reissner—Nordström black hole.

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1. THE PROBLEM

Let (M_1, g_1) be some Riemannian manifold; assume that $f_1 : M_1 \to \mathbb{R}$ is a smooth positive function, $M_0 = \mathbb{R}$, and $g_0 = dt^2$ is the standard Riemannian metric on M_0. Let us consider $M = M_0 \times M_1$, $f = f_1 \circ p_1$, and $g = -f p_0^* g_0 + p_1^* g_1$, where $p_0 : M \to M_0$ and $p_1 : M \to M_1$ are natural projections. Then (M, g) is a Lorentzian manifold which at the same time is the twisted product of manifolds $(M_0, -g_0)$ and (M_1, g_1) with the twisted function f_1 ([1], P. 59). This manifold possesses a natural time orientation, i.e., there exists a smooth vector field $\overline{X}_0$ in M which meets conditions $dp_0(\overline{X}_0) = 1$ and $dp_1(\overline{X}_0) = 0$. Let us fix this orientation.

Now let us consider a closed 2-form F_1 on M_1 and put $F = p_1^* F_1$ and $u \equiv 1/2$. According to [2] and [3], the quadruple $\Gamma = (M, g, F, u)$ is a relativistic type gyroscopic system. Its motion is given by future directed solutions of the following system:

$$\frac{\nabla}{ds}\left(\frac{dx}{ds}\right) = F'\left(\frac{dx}{ds}\right), \tag{1}$$

$$g\,(dx/ds,\, dx/ds) = -1; \tag{2}$$

here ∇ is the covariant derivative operator in (M, g), and F' is the linear operator field on M defined by the correlation $g(F'(\overline{X}), \overline{Y}) = F(\overline{X}, \overline{Y})$.

We are interested in motions $x : [0, \beta] \to M$ of the system Γ, $\beta > 0$, such that

$$x(0) = v, \quad x(\beta) = w; \tag{3}$$

here v is an arbitrary point in the manifold M, and w is the point from $I^+(v)$, i.e., from the chronological future of the point v.

Note that the form of gyroscopic forces F is not necessarily exact. Moreover, in the main Theorem 2 we assume the existence of partially-smooth spheroids in M such that integrals on F over these spheroids do not vanish. In this case the action functional of the system Γ is multivalued [4, 5].

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