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3D Circular Shapes and Curve Skeletons

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Abstract—The medial axis of a planar shape is the set of points having at least two closest points on the shape boundary. This notion is widely used in computer science. In this paper we propose a mathematical model enabling us to define the notion of a curve skeleton as a 3D generalization of the 2D medial axis. We propose a criterion for comparing various particular methods for the construction of curve skeletons.

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1. INTRODUCTION

Let $\widetilde{\Omega}$ be an open bounded n-dimensional subset of $\mathbb{R}^n$ with the boundary $\partial\Omega$. Denote by

$$\Omega = \widetilde{\Omega} \cap \partial\Omega$$

the closure of this set. We call this closure a figure.

Definition 1. The medial axis of the open set $\widetilde{\Omega}$ is the set $\mathcal{M} \subset \Omega$ of points which have at least two closest points on the boundary of Ω, i.e.,

$$\mathcal{M} = \{\mathbf{a} \in \Omega \mid \exists \mathbf{x}, \mathbf{y} \in \partial\Omega : \mathbf{x} \neq \mathbf{y},\ \rho(\mathbf{a}, \mathbf{x}) = \rho(\mathbf{a}, \mathbf{y})\ \forall \mathbf{z} \in \partial\Omega\ \ \rho(\mathbf{a}, \mathbf{z}) \geq \rho(\mathbf{a}, \mathbf{x})\}.$$

Definition 2. The medial axis of the figure Ω is the closure of the medial axis of the open set $\widetilde{\Omega}$.

The notion of the medial axis was introduced in [1]. In a general case, the medial axis of an n-dimensional figure contains $(n-1)$-dimensional subsets. There always exists a homotopy equivalence between a figure and its medial axis [2]. The notion of the medial axis allows generalizations for n-dimensional figures which, in general, differ from subsets of $\mathbb{R}^n$ [3].

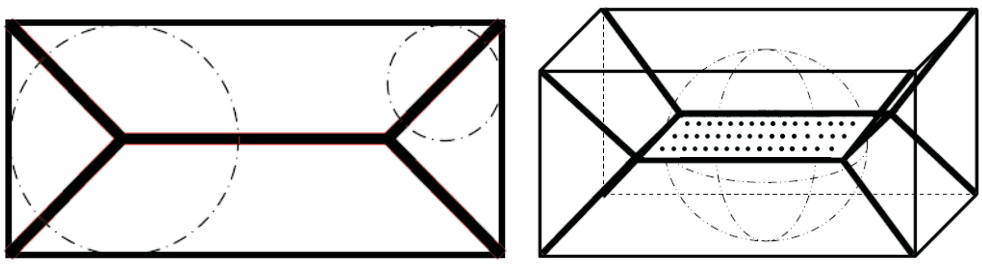

Fig. 1. The medial axis of a 2D rectangle and a 3D parallelepiped.

The medial axis of a 2D figure is a planar graph; it is also called the skeleton of this figure. Properties of the skeleton of a plane figure depend on the basic metric and topological properties of this figure. It is

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