Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 45, No. 2, pp.30–35, 2001 *UDC 517.544*

PARAMETRIC REPRESENTATION OF COMPOSITION FACTORS IN THE THEOREM ON FACTORIZATION

V.V. Kozhevnikov

1. Let Γ stand for a certain bounded closed subset of the w-plane. By $\Sigma(\Gamma)$ we shall understand the class of functions meromorphic and univalent in the complement to the set Γ, which in a neighborhood of infinity possess the decomposition

$$T(w) = w + a_0 + \frac{a_1}{w} + \frac{a_2}{w^2} + \cdots ;$$

here dots stand for terms containing greater powers of $1/w$.

The following theorem is known.

Theorem 1 (on factorization). *Let a function $w = w(z)$, regular and schlicht in the ring $r < |z| < R$, map conformally this ring onto a doubly-connected domain D, whose complement to the whole w-plane consists of the two non-intersecting components: A bounded continuum Γ and an unbounded continuum B. Moreover, let the continuum Γ correspond to the interior boundary component $|z| = r$ of the ring $r < |z| < R$ in the following sense: When a point z of the ring tends to its interior boundary component, the point $w = w(z)$ of the domain D tends to the continuum Γ.*

Then the unique function $T_w(w)$ of the class $\Sigma(\Gamma)$ exists such that the composition $\Phi_w(z) = T_w \circ w(z)$, defined in the ring $r < |z| < R$, can be analytically continued into the complete disk $|z| < R$, $\Phi_w(0) = 0$, and in this disk is a regular and univalent function.

V.D. Yerokhin (see [1], [2]) was first who established (and in more general form) the theorem on factorization. This theorem arises from the requirements of the theory of approximation. Later the ideas exposed by V.D. Yerokhin were developed by other mathematicians (see [3]–[5]).

The theorem on factorization, which is a typical existence theorem, does not allow to construct the functions $T_w(w)$ and $\Phi_w(z)$. To obtain a constructive version of this theorem one can use various approaches. In this article we use the approach close to Löwner's parametric approach in [6].

Let, as above, a regular function $w = w(z)$ map conformally the ring $r < |z| < R$ onto a doubly-connected domain D.

Definition. A univalent function $v = v(w, \tau)$ of the complex variable $w \in D$ and the real variable τ, $0 < \tau < \tau^*$, is called a univalent variation of the function $w = w(z)$ if

a) for each value of τ, $0 < \tau < \tau^*$, as a function of variable w, $v(w, \tau)$ is schlicht in the domain D;