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Nonlinear Summation of Power Series and Exact Solutions of Evolution Equations

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Abstract—The technique of quadratic and cubic summation of power series in the perturbation method was first used for finding exact solutions to nonlinear evolution equations. The series were construction with the use of exponential partial solutions to linearized equations. The solution of both classic and modified nonintegrable Korteweg-de Vries equations, the modified Burgers equation, and the Fisher one allows one to demonstrate specific features of the mentioned method. We obtain exact solitary wave solutions to the mentioned equations in the form of a wave impulse and a wave front and show that summation parameters depend on the pole orders of the desired solutions.

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INTRODUCTION

The development of new analytical methods for solving nonlinear partial differential equations remains actual for several decades. When seeking for a solution by expanding the dependent variable into a power series, in particular, by the perturbation method, the key moment consists in the choice of the series summation method. Usually one uses rational and quadratic approximations of power series [1] as approximate values of their sums. In [2] one proves the possibility of the quadratic approximation of elliptic integrals and trigonometric functions.

In this paper, we propose a new method for constructing exact solitary wave solutions to nonlinear evolution equations. This method is based on the use of linear, quadratic, or cubic sums of power series. The method consists of two steps. On the first step, one constructs the series of the perturbation method for the equation under consideration. On the second step, one applies a proper summation technique to the constructed series and thus obtains an exact solution to the equation.

1. NONLINEAR SUMMATION OF POWER SERIES

Consider a power series $u(z) = \sum\limits_{n=0}^{\infty} a_n z^n$ with known coefficients a_n and a system of complete polynomials P_k^M of degree M with arbitrary coefficients

$$P_k^M = p_0^{(k)} + p_1^{(k)} z + p_2^{(k)} z^2 + \cdots + p_M^{(k)} z^M.$$

Grouping the expression

$$\sum_{k=0}^{N} P_k^M u^k \qquad (1)$$

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