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Brunn–Minkowski Type Inequalities for Conformal and Euclidean Moments of Domains

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Abstract—We prove Brunn–Minkowski type inequalities for three new functionals which are power moments for conformal and Euclidean characteristics of domains.

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1. INTRODUCTION

There are many papers about Brunn–Minkowski type inequalities in Geometrical Analysis, Mathematical Physics and Probability Theory. We only cite some papers which present the important steps in the development of this part of the theory of inequalities.

In 1956 H. Hadwiger [1] discovered that Brunn–Minkowski type inequalities are true for two inertia moments of convex domains, namely, for the inertia moment about the center of mass and the inertia moment about a hyperplane.

In 1971–72 A. Prékopa [2] and L. Liendler [3] proved the following functional version of the Brunn–Minkowski inequality.

Theorem A. *Let $0 < t < 1$, and let φ_0, φ_1, h be nonnegative integrable functions in the Euclidean space $\mathbb{R}^n$. If for all $x, y \in \mathbb{R}^n$*

$$h((1-t)x + ty) \geq \varphi_0(x)^{1-t}\varphi_1(y)^t, \tag{1}$$

then

$$\int_{\mathbb{R}^n} h(x)dx \geq \left(\int_{\mathbb{R}^n} \varphi_0(x)dx\right)^{1-t}\left(\int_{\mathbb{R}^n} \varphi_1(x)dx\right)^t. \tag{2}$$

One can find improvements of Theorem A and some new results in the papers by H. J. Brascamp, E. H. Lieb [4] and C. Borell [5].

The excellent survey papers by R. J. Gardner [6] and by F. Barthe [7] contain the literature and the main results on the Brunn–Minkowski type inequalities before 2006. We also note some recent papers [8–10].

In 2007 G. Keady [11] established a new Brunn–Minkowski type inequality for the geometrical functional defined in [12]. In this paper we develop Keady's result.

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