ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2009, Vol. 53, No. 8, pp. 58–59. © Allerton Press, Inc., 2009.
Original Russian Text © D.V. Reshetnikov, 2009, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2009, No. 8, pp. 71–72.

Computation of Cohomology Groups of the Lie Algebras of Type B_n and C_n

D. V. Reshetnikov[1]*

[1] *Nizhni Novgorod State University, pr. Gagarina 23, Nizhni Novgorod, 603950 Russia*

Received April 23, 2007

Abstract—We adduce the results of the numerical experiment in calculating the spaces of local deformations of classical Lie algebras of type B_n and C_n in characteristic $p = 2$.

DOI: 10.3103/S1066369X0908009X

Key words and phrases: *cohomology group, classical Lie algebra, local deformation, characteristic 2.*

The classical Lie algebras over fields of zero characteristic and the characteristic $p > 3$ are rigid [1]. Over a field of characteristic 2 and 3 the classical Lie algebras may have nontrivial deformations. See papers [2, 3] for the description of global deformations of the Lie algebra C_2. In [4] and [5] the rigidity of the classical Lie algebras over a field of characteristic $p > 2$ was proved for all types except the algebra of type C_2 for $p = 3$. The spaces of local deformations (the second cohomology group with coefficients in the adjoint module) of the classical Lie algebras with a simply laced root system over a field of characteristic $p = 2$ were found in [6]. The computation of local deformations of the remaining classical Lie algebras presents a big difficulty.

A software has been developed to compute the cohomology $H^n(\mathfrak{g}; A)$ of an arbitrary finite dimensional Lie algebra $\mathfrak{g}$ with coefficients in a finite dimensional $\mathfrak{g}$-module A for the case of the given structural constants of $\mathfrak{g}$ and A. In addition, for a graded Lie algebra $\mathfrak{g}$ and a graded module A the computations can be simplified by taking into account the grading on the cohomology. This software has been used to compute the spaces of local deformations of some classical Lie algebras of characteristic $p = 2$.

The results of the numerical experiment for the cohomology of the classical Lie algebras of type C_n, $3 \leqslant n \leqslant 8$, have shown that the dimension of the cohomology group $H^2(\mathfrak{g}; \mathfrak{g})$ is equal to $2n^2 + 5n - 1$ where n is the rank of $\mathfrak{g}$. A basis is formed by cocycles of the following weights (in the notation of [7]):

1. for $\mu = 0$ we have $\dim H^2_\mu(\mathfrak{g}; \mathfrak{g}) = n - 1$;
2. for $\mu = \pm 2\varepsilon_i$, $1 \leqslant i \leqslant n$, we have $\dim H^2_\mu(\mathfrak{g}; \mathfrak{g}) = 3$;
3. for $\mu = \pm 2(\varepsilon_i \pm \varepsilon_j)$, $1 \leqslant i < j \leqslant n$, we have $\dim H^2_\mu(\mathfrak{g}; \mathfrak{g}) = 1$.

The results of the numerical experiment for the cohomology of the classical Lie algebras of type B_n, $3 \leqslant n \leqslant 7$, have shown that $H^2(\mathfrak{g}; \mathfrak{g}) = 0$ when $n \neq 4$. For the Lie algebra of type B_4 the second cohomology group has dimension 16 with a basis formed by cocycles of weights $\mu = \pm\varepsilon_1 \pm \varepsilon_2 \pm \varepsilon_3 \pm \varepsilon_4$ where signs are chosen arbitrarily. It is easy to see that all these weights are conjugate under the action of the Weyl group, so that it suffices to look at a cocycle of one single weight. Computations show that the cocycle of weight $\mu = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4$ has image in the center of the algebra and takes the form

$$\varphi_\mu = X_{-\varepsilon_1-\varepsilon_2} \wedge X_{-\varepsilon_3-\varepsilon_4} \otimes H_{\varepsilon_4} + X_{-\varepsilon_1-\varepsilon_3} \wedge X_{-\varepsilon_2-\varepsilon_4} \otimes H_{\varepsilon_4} + X_{-\varepsilon_1-\varepsilon_4} \wedge X_{-\varepsilon_2-\varepsilon_3} \otimes H_{\varepsilon_4}.$$

The remaining cocycles in the basis are obtained from φ_μ by the action of the Weyl group [8].

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, grant 05-01-00580.

*E-mail: genserg@hotmail.com.