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Varieties of Rings, Where All Subdirectly Irreducible Finite Rings are Armendariz Ones

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Abstract—In this paper we study varieties of rings, where all subdirectly irreducible finite rings are Armendariz. We also describe the locally finite varieties of Armendariz rings.

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In this paper we study associative rings.

Definition 1. A ring R is called *Armendariz* if any polynomials $f(x) = a_0 + a_1x + \cdots + a_mx^m$ and $g(x) = b_0 + b_1x + \cdots + b_nx^n \in R[x]$ such that $f(x)g(x) = 0$ satisfy the equalities $a_ib_j = 0$ with all $i = 0, 1, \ldots, m$ and $j = 0, 1, \ldots, n$.

Definition 2. A ring R is called *weak Armendariz* if any polynomials $f(x) = a_0 + a_1x$ and $g(x) = b_0 + b_1x \in R[x]$ such that $f(x)g(x) = 0$ satisfy the equalities $a_ib_j = 0$ with $i = 0, 1$ and $j = 0, 1$.

In 1974 E. Armendariz proved [1] that if the product of two polynomials whose coefficients belong to a ring without nonzero nilpotent elements equals zero, then all possible pairwise products of coefficients of these polynomials equal zero. In 1997 in the paper [2] rings with this property were called "Armendariz".

In 1998 D. Anderson and V. Camillo proved [3] that a ring of polynomials over an Armendariz ring with identity is Armendariz and that an Armendariz regular (in the sense of von Neumann) ring with identity has no nonzero nilpotent elements.

In [4] one proved that all idempotents of an Armendariz ring with identity are central and for any ring R with identity that has no nonzero nilpotent elements the ring

$$S = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \;\middle|\; a, b, c, d \in R \right\}$$

is also Armendariz. Later in [5] one proved the converse assertion: If a ring S defined (as indicated above) for a certain ring R with identity is Armendariz, then R has no nonzero nilpotent elements.

In 2002 the following assertions were proved [6]:

1) if the factor ring R/I of a ring R with identity is Armendariz for a certain ideal I without nonzero nilpotent elements, then the ring R is Armendariz;

2) if a ring R with identity has the complete classical right ring of quotients Q, then R is Armendariz if and only if so is the ring Q.

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