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Unconditionally Stable Schemes for Convection–Diffusion Problems

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Abstract—Convection–diffusion problems are basic ones in continuum mechanics. The main features of these problems are connected with the fact that their operators may have an indefinite sign. In this paper we study the stability of difference schemes with weights for convection–diffusion problems where the convective transport operator may have various forms. We construct unconditionally stable schemes for nonstationary convection–diffusion equations based on the use of new variables. Similar schemes are also used for parabolic equations where the operator represents the sum of diffusion operators.

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1. INTRODUCTION

In applied modeling of problems of continuum mechanics, convection–diffusion equations [1–3] are basic ones. Their main features are, in particular, the non-self-adjointness of the problem operator and the predominance of the convection transfer over the diffusion one in many applied problems. In convection–diffusion problems for compressible media the problem operator is sign-definite. In this case the process under consideration may appear to be non-dissipative, i.e., the norm of the solution to the homogeneous problem may be nondecreasing with respect to time. It is necessary to keep such behavior of the solution norm on the discrete level when choosing time approximations.

The numerical solution of nonstationary problems for convection–diffusion equations is most often based on two- and three-level schemes. One can study the stability and convergence of approximate solutions with the help of general results of the stability (well-posedness) theory of operator-difference schemes obtained by A. A. Samarskii [4–6]. It should be noted that because of the non-self-adjointness of operators, the direct application of general stability criteria in convection–diffusion problems may be difficult. Note also that because of the fact that the problem operator is not sign-definite, we have to apply only ϱ-stable ($\varrho > 1$) operator-difference schemes. For solving nonstationary problems on large time intervals it is better to use asymptotically stable schemes [5, 7]. Such schemes guarantee the desired behavior of the solution with the separation of its main harmonic on large time intervals and damping of other ones (the regular mode for the heat conductivity equation [8]).

The standard schemes used in computational practice need some correction even when solving dissipative problems. For example, neither the ordinary purely implicit (Euler) scheme, nor the symmetric (Crank–Nicholson) one gives an exact solution to the test problem ($\lambda > 0$)

$$\frac{du}{dt} + \lambda u = 0, \quad u(0) = u^0.$$

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