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A NEW APPROACH TO THE THEORY OF LINEAR PROBLEMS FOR SYSTEMS OF DIFFERENTIAL PARTIAL EQUATIONS, I

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Introduction

In a series of three articles we suggest a new concept of a solution of a linear problem and, basing on this concept, we develop the theory of resolvability. In doing so we shall use the following notation:

$\mathbb{C}^n \supset \mathbb{R}^n \supset \mathbb{Z}^n \supset \mathbb{M}^n$ are sets of all n-dimensional vectors, whose all coordinates are complex, real, integer, and integer nonnegative numbers, respectively;

in the case where $z \in \mathbb{C}^n$ (analogously, $\mathbb{R}^n$, $\mathbb{Z}^n$, $\mathbb{M}^n$) we have $|z| = (|z_1|^2 + \cdots + |z_n|^2)^{1/2}$;

$x \le y$ ($x < y$) for $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$ if, for all coordinates, we have $x_j \le y_j$ ($x \le y$ and $x \ne y$);

$z^\alpha \equiv z_1^{\alpha_1} \cdots z_n^{\alpha_n}$ $\forall z \in \mathbb{C}^n$, $\forall \alpha = (\alpha_1, \dots, \alpha_n) \in \mathbb{M}^n$; $z_j^0 \equiv 1$;

Φ is a finite set of multi-indices, i. e., elements from $\mathbb{M}^n$;

an element $y \in \Phi$ is said to be maximal if there is no $x \in \Phi$ for which $x > y$;

$L^2_\nu(\Omega)$ is a set of all Borel measurable ν-dimensional functions $u(x)$, which satisfy the condition $\int\limits_\Omega |u(x)|^2 dx < +\infty$;

if Ω is an open domain in $\mathbb{R}^n$, then $\mathbb{D}'(\Omega, \nu)$ is the set of all vectors $u(x) = (u_1(x), \dots, u_\nu(x))$, where each coordinate is a generalized function;

$W^{\Phi,2}_\nu(\Omega)$ is a space of Sobolev type, i. e., a set of all $u(x) \in \mathbb{D}'(\Omega, \nu)$ for which the generalized derivatives $u^{(\alpha)}(x) = (u_1^{(\alpha)}(x), \dots, u_\nu^{(\alpha)}(x))$ for all $\alpha \in \Phi$ belong to $L^2_\nu(\Omega)$;

$C^\Phi_\nu(\Omega)$ is a set of all ν-dimensional functions which are continuously differentiable in Ω up to the orders $\alpha \in \Phi$, whose existing derivatives are bounded in Ω;

$C^\Phi_\nu(b-a)$ (here and below $a \in \mathbb{R}^n$, $b \in \mathbb{R}^n$) is a subset from $C^\Phi_\nu(\mathbb{R}^n)$ of all functions which are $(b-a)$-periodic along with existing derivatives; as soon as all coordinates of $(b-a)$ equal 2π, instead of $C^\Phi_\nu(b-a)$ we shall write $C^\Phi_\nu(2\pi)$;

$L^2_\nu(b-a)$ is a set of all Borel measurable ν-dimensional, $(b-a)$-periodic functions $u(x)$, which satisfy the condition $\int\limits_{[a,b]} |u(x)|^2 dx < +\infty$. In addition, $[a,b] = \{x : a \le x \le b\}$, the latter integral and the notation $\int\limits_a^b |u(x)|^2 dx$ have the same meaning;

$W^{\Phi,2}_\nu(b-a)$ stands for a set of all $u(x) \in \mathbb{D}'(\Omega, \nu)$ for which $u^{(\alpha)}(x) \in L^2_\nu(b-a)$ with any $\alpha \in \Phi$;

$$C^{+\infty}_\nu(\Omega) = \cap C^\Phi_\nu(\Omega), \qquad C^{+\infty}_\nu(b-a) = \cap C^\Phi_\nu(b-a),$$
$$W^{+\infty,2}_\nu(\Omega) = \cap W^{\Phi,2}_\nu(\Omega), \quad W^{+\infty,2}_\nu(b-a) = \cap W^{\Phi,2}_\nu(b-a);$$

in addition, the intersection is taken over all $\Phi \subset \mathbb{M}^n$.